

# EFFICIENT DISCLOSURE LAWS\*

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## Abstract

If information can be disclosed voluntarily for a cost, a commitment to a law mandating disclosure over certain events is socially desirable. The efficient law takes the form of a threshold such that only unfavorable events are subject to mandatory disclosure. With single-peaked distributions, we establish that the mandatory (voluntary) disclosure threshold is (above) below the mode of the distribution, implying that the most likely event is never disclosed in equilibrium. In the case of symmetric distributions, the market reaction to any voluntary disclosure is positive while the price reaction to mandatory disclosure or non-disclosure is negative.

**Keywords:** Mandatory; voluntary; standards; law; efficiency.

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Various laws mandate the disclosure of information prior to a sale. For example, public firms are required to supply financial statements as well as special regulatory filings (Coffee 1984, Zeff 2003, Dranove and Jin 2010). Pharmaceutical companies cannot sell drugs without some testing and some disclosure over the results of such tests and potential side effects (Ma, Marinovic and Karaca-Mandic 2015). In these settings, mandatory and voluntary disclosures coexist but appear to serve different functions. However, the theoretical foundation for mandatory disclosure in the voluntary disclosure is not fully understood, as voluntary disclosure models tend to exhibit *over*-provision of information (Dye 1986, Shavell 1994). If buyers are price protected, sellers should have enough incentives to disclose information voluntarily if the effect of that information on price is superior to the cost of providing it.

This paper speaks to this puzzle in a model in which firms have access to a costly certification technology (Verrecchia 1983, Levin, Peck and Ye 2009, Marinovic and Sridhar 2015), that is, information can be disclosed for a cost. We ask three questions. When do sellers benefit ex-ante, i.e., sell at a higher expected price, when bound to some mandatory disclosure before receiving their information? Are there any characteristics of optimal mandatory disclosures that make them different from voluntary disclosure? And, what are determinants of the desirable scope for mandatory disclosure?

We demonstrate that laws that require disclosing all sufficiently unfavorable news can increase the expected selling price; surprisingly, this result holds even in a pure-exchange economy with risk-neutral sellers, in which information serves no productive purpose and more mandatory disclosure would only increase disclosure costs (holding voluntary disclosure constant). Thus, our optimality result is not driven by the specification of a decision problem with a particular value of information (Demski 1973). Voluntary disclosure creates a role for mandatory disclosure in that it reduces the level of socially-excessive voluntary disclosure.

If the distribution of the privately-observed news is single-peaked, the mandatory dis-

closure threshold is below the mode, while the voluntary disclosure is above the mode. Mandatory disclosure cuts the the left tail of the posterior following a non-disclosure, which tends to have a positive effect on the non-disclosure expectation. This also implies that, in an efficient law, the most likely (modal) event is never disclosed. If, in addition, the distribution is symmetric, the unconditional probability of mandatory disclosure is smaller than the unconditional probability of voluntary disclosure, and both mandated disclosure and voluntary withholding imply negative price reactions.

This result generalizes to environment in which information serves post-sale productive purposes. Post-sale decision making or production renders the price strictly convex in information, which favors more dispersed posterior beliefs. The socially-preferred regime would feature disclosure over both sufficiently favorable and sufficiently unfavorable disclosure. When voluntary disclosure is allowed, sellers over-invest in disclosing good news relative to this social optimum, so no further mandatory disclosure is required for good news. An efficient law mandating the disclosure of bad news now serves two purposes: a reduction in excess voluntary disclosure which is present under pure-exchange and, with production, the supply of useful bad news for productive purposes. By contrast, if the seller is risk-averse, which renders a strictly concave pricing function, mandating disclosure unfavorable events may not be efficient.

**Literature review.** The paper most closely to ours is Jiang and Yang (2016), which provides a similar result in a different signalling environment. In their model, the firm commits to an information system subject to an entropy constraint and then engages in costly signalling. Their main result is that the optimal information system provides information about bad news. In their model, mandatory disclosure is constrained but not costly within this constraint. In comparison, our focus here is on explaining the existence of mandatory disclosure, so we use reporting technologies whose costs are similar should reports be mandatory or voluntary.

To our knowledge, few studies have shown that mandatory disclosures are desirable in

the presence of voluntary disclosure. The classic early study of Shavell (1994), for example, shows that information acquisition is distorted in the presence of voluntary disclosure and, more recently, Ben-Porath, Dekel and Lipman (2014) show that voluntary disclosure systematically distorts ex-ante project selections; these studies speak primarily to the potential benefits of direct regulation of the decisions that come prior to the disclosure decision.

Many other studies have analyzed interactions between mandatory and voluntary channels. In Einhorn (2005), the nature of the mandatory disclosure can change the interpretation of a given voluntary disclosure signal as good or bad news. In Gao (2013), the manager has access to a given set of distribution of reports and will choose the reports as a function of the binary classification imposed by the standard. In Bertomeu and Magee (2015), voluntary disclosure reduces political demands for more mandatory disclosure. The substitution between voluntary and mandatory disclosure is also implicit in Verrecchia (1990), in that a more informative information environment (due to say, more transparent mandatory disclosures) is equivalent to a reduction in the dispersion of the manager's private information and thus would cause less disclosure for a given cost.<sup>1</sup>

The literature in which mandatory and voluntary disclosure may interact is much broader, XXXX

There is also a large literature on costly disclosure that we cannot review here in its entirety, see, e.g., Beyer, Cohen, Lys and Walther (2010), and which applies the model Verrecchia (1983) in financial reporting. As recent examples, Jorgensen and Kirschenheiter (2003) examine the effect of disclosure on the cost of capital and Marinovic and Varas (2014) solve a dynamic disclosure model in which disclosing good information or failing to disclose bad information may involve a cost; in this latter example, the scope of regulation

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<sup>1</sup>While we only discuss a few examples here, the literature in which regulations and voluntary disclosure interact is much broader and the object of many recent studies, see, e.g., Gigler and Hemmer (2001), Langberg and Sivaramakrishnan (2008), Williams, Hughes and Levine (2010), Beyer and Guttman (2012), Edmans, Heinle and Huang (2016) or Chen and Jorgensen (2016).

is in terms of the litigation cost or enforcement.<sup>2</sup>

Lastly, an extensive prior literature has focused on the social benefits induced by product market externalities, see Vives (1984), Wagenhofer (1990), Admati and Pfleiderer (2000) or Suijs and Wielhouwer (2014). In these models, mandatory disclosure can increase the efficiency of production at the industry level or reduce the deadweight losses created by oligopolistic competition. Since the product market channel has been analyzed in this prior literature, we have abstracted away from these benefits and only characterize the demand that comes exclusively from the capital market. In our model, the demand for mandatory disclosure is caused only by the price externality of voluntary disclosure.

## 1. The Model

For expositional purposes, we use the analogy of a single seller of an asset in a competitive market. The timeline is as follows. At date 1, the seller privately observes a signal regarding the value of the asset  $\tilde{x}$ , drawn from a distribution with continuously differentiable p.d.f.  $f(\cdot)$  and c.d.f.  $F(\cdot)$  on support  $\mathbb{R}$  with finite mean. Further, assume that p.d.f.  $f(\cdot)$  is strictly logconcave with sub-exponential lower tail.<sup>3</sup> We denote the (unique) mode of the distribution  $m \in \mathbb{R}$ .

At date 2, a public signal  $r(x) \in \{x, nd\}$  is issued, where  $r(\cdot)$  is a function such that  $r(x) = x$  is interpreted as a verifiable report that  $\tilde{x} = x$  and  $r(x) = nd$  is a non-disclosure. At date 3, conditional on a report  $r$ , the asset is sold for a price  $P(r)$  and the seller achieves a payoff  $P(r) - c\mathbf{1}(r \neq nd)$  net of a verification cost  $c > 0$  (Jovanovic 1982, Verrecchia 1983).<sup>4</sup>

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<sup>2</sup>There is also a growing literature on voluntary disclosure and certification in dynamic settings, see, e.g., Acharya, Demarzo and Kremer (2011) or Guttman, Kremer and Skrzypacz (2014).

<sup>3</sup>The purpose of this assumption is to make sure that when the mandatory disclosure set does not truncate the lower portion of the distribution, there always exist voluntary disclosure equilibrium which is not unravelling. However, we will show that the efficient mandatory disclosure set would always truncate the lower portion of the distribution. Hence the main result does not hinge on this assumption.

<sup>4</sup>The cost is sometimes interpreted as a proprietary cost, borne by the firm but not necessarily socially, in which case some other party is gaining from disclosure (e.g., competitors, suppliers, consumers, etc.).

The price  $P(r)$  is a function of buyers' rational expectation about value of the asset given his information set. For now, we assume  $P(x) = x$ , and, when possible,  $P(nd) = \mathbb{E}(\tilde{x}|r(\tilde{x}) = nd)$ .<sup>5</sup> The reporting function  $r(\cdot)$  is determined by two disclosure channels. First, a law prescribes that all  $x \in D_m$  must be reported, where  $D_m$  is a finite union of non-adjacent open intervals.<sup>6</sup> Hence  $r(x) = x$  if  $x \in D_m$ . Second, for any  $x \notin D_m$ , the seller chooses to report voluntarily or withhold, so that  $r(x) \in \{x, nd\}$  maximizes  $P(r) - c\mathbf{1}(r \neq nd)$  where, to save space, we assume that the seller withholds when indifferent.

The next definition is a straightforward formulation of a voluntary disclosure equilibrium in the presence of mandatory disclosure.

**Definition 1** *An equilibrium is  $\mathcal{E}(D_m) = (P, r)$  with two functions  $P : \mathbb{R} \cup \{nd\} \rightarrow \mathbb{R}$  and  $r : \mathbb{R} \rightarrow \mathbb{R} \cup \{nd\}$  that satisfy:<sup>7</sup>*

- (i) *the seller withholds when it is in accordance with the law and would yield a higher price, that is,  $r(x) = nd$  if and only if  $x \notin D_m$  and  $P(nd) \geq P(x) - c$ ;*
- (ii) *whenever possible, Bayes rule applies, that is,  $P(y) = \mathbb{E}(\tilde{x}|r(\tilde{x}) = y)$ .*

Our objective will be to show the existence of an efficient disclosure law, that is, laws that improves the ex-ante surplus of the seller, and characterize its properties.<sup>8</sup> Further, we are interested in situations in which the information would not be disclosed voluntarily if it were not subject to the law, as noted next.

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Under the strong assumption that costs are fully redistributive, a regulation prescribing full-disclosure would be optimal so that a critical assumption for our analysis is that costs carry some deadweight loss that is not internalized by the regulator. This could, for example, represent verification costs or value transfers to parties not represented by the regulator (e.g., foreign firms or investors).

<sup>5</sup>We discuss the implication of non-linear pricing in the extension.

<sup>6</sup>If two open intervals are adjacent, they can be joined to be a single interval.

<sup>7</sup>The restriction to  $P(nd)$  rules out unravelling equilibria by assumption. However, Definition 2 would never select a combination of mandatory disclosure law and unravelling, since this would never maximize the surplus of the seller.

<sup>8</sup>Alternatively, the model can be interpreted as that there exists a continuum of sellers and the law maximizes the total surplus.

**Definition 2**  $D_m$  is an efficient disclosure law if there exists an equilibrium  $\mathcal{E}(D_m) = (P, r)$  such that

(i) for any other law  $D_m^a$  and equilibrium  $\mathcal{E}(D_m^a) = (P^a, r^a)$ ,  $\mathbb{E}[P(r(\tilde{x})) - c\mathbf{1}(r(\tilde{x}) \neq nd)] \geq \mathbb{E}[P^a(r^a(\tilde{x})) - c\mathbf{1}(r^a(\tilde{x}) \neq nd)]$ ;

(ii) for any  $x$  such that  $P(x) - c > P(nd)$ , it holds that  $x \notin D_m$ .

Part (i) states that a disclosure law can lead to a higher total surplus relative to any other equilibrium that could be achieved with a different law. Disclosure games with costly disclosure can have multiple equilibria but there is always a unique equilibrium that is weakly preferred by all sellers regardless of their information. By construction, the efficient disclosure law must attain the highest surplus within this preferred equilibrium.

Part (ii) is imposed so that we label mandatory disclosure set as a binding legal requirement that would not have been adopted by sellers if they had not been required to. This part implies that we can (hereafter) restrict the attention to  $\sup D_m < \infty$  since all sufficiently favorable events will be disclosed voluntarily; for expositional purpose, we maintain this restriction throughout in the rest of our analysis.

We point next to a few key aspects of our research question and assumptions:

- *Seller regulation preference.* Our analysis pertains only to seller-preferred regulations, as we do not explicitly model the welfare consequences to other parties that do not make a strategic decision. In the special case of a perfectly competitive market with homogenous price-protected buyers, e.g., if competitive buyers price the asset at its expected value, buyers would be indifferent to disclosure; but, more generally, the effect of information on welfare has been the object of an extensive prior literature (Dye 1990, Wagenhofer 1990, Fishman and Hagerty 2002, Suijs and Wielhouwer 2014).<sup>9</sup>

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<sup>9</sup>Depending on the market structure, buyers may be ambiguously affected by disclosure either because of product market consequences (Wagenhofer 1990, Suijs and Wielhouwer 2014) or even adversely when the buyers are price-protected but risk-averse (Dye 1990).

- *Perfect enforcement.* We focus only on the disclosure choice (which events will be reported) and assume that the law is perfectly enforceable. This is also an important reason why we focus on costly disclosure because voluntary disclosure models in which the seller may claim to be uninformed (Dye 1985, Jung and Kwon 1988) require more structure on the ex-post verification game which should feature an interaction between disclosure and enforcement (Dye 2011).
- *Disclosure technology.* We abstract from any technological advantage or disadvantage of voluntary disclosure, and assume that the cost incurred is the same for both disclosure channels. This is not intended for realism, in that in practice mandatory may be more or less costly to make than voluntary disclosure, but to make a fair conceptual comparison between the two modes of disclosure.

## 2. Main result

As a first step to the analysis, it is convenient to rewrite the voluntary disclosure disclosure strategy in terms of a function above which news is voluntarily disclosed. However, a complication that arises endogenously in our setting is that  $\tilde{x}|\tilde{x} \notin D_m$ , the set of events that are not subject to mandatory disclosure, may not be logconcave even if  $\tilde{x}$  is logconcave so that a voluntary disclosure equilibrium is not unique. As noted in Definition 2, we select the equilibrium with the lowest expected disclosure cost; this is made formal below.

Given an arbitrary mandatory disclosure set  $D_m$ , an equilibrium  $\mathcal{E}$  features a voluntary disclosure threshold  $\tau(D_m|\mathcal{E}) \in \mathbb{R}$  such that all sellers with  $x > \tau(D_m|\mathcal{E})$  would voluntarily disclose if not required to disclose. Since it is optimal to disclose if and only if  $x - c > P_{D_m,\mathcal{E}}(nd)$ , where  $P_{D_m,\mathcal{E}}(nd)$  is the non-disclosure price in  $\mathcal{E}$  given  $D_m$ , one can rewrite this threshold in terms of the nondisclosure price as  $\tau(D_m|\mathcal{E}) - c = P_{D_m,\mathcal{E}}(nd)$  (Jovanovic

1982, Verrecchia 1983), that is,

$$\tau(D_m|\mathcal{E}) - c = \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq \tau(D_m|\mathcal{E})). \quad (2.1)$$

The fact that  $\tilde{x}$  has sub-exponential lower tail implies that at least one solution to this equation exists as long as  $D_m \neq \mathbb{R}$ . If the solution  $\tau(D_m|\mathcal{E})$  is unique, we denote it as  $\tau(D_m)$ . If the solution is not unique, note that the highest threshold  $\tau(D_m|\mathcal{E})$  is associated with the lowest expected disclosure cost. Hence, in Definition 2, an efficient law must achieve the lowest disclosure cost in this equilibrium. With a slight abuse in notation, we denote  $\tau(D_m) = \sup_{\mathcal{E}} \tau(D_m|\mathcal{E})$  as the seller-preferred voluntary disclosure threshold.

**Lemma 1**  $\tau(D_m) = \max_{\mathcal{E}} \tau(D_m|\mathcal{E}) \in \mathbb{R}$ , for any  $D_m \neq \mathbb{R}$ .

The ex-ante surplus of the seller  $\mathcal{W}(D_m) = \mathbb{E}[P(r(\tilde{x})) - c\mathbf{1}(r(\tilde{x}) \neq nd)]$  is defined as the expected price net of disclosure costs. Then, the law of iterated expectations implies that the expected price is the expected news  $\mathbb{E}[P(r(\tilde{x}))] = \mathbb{E}(\tilde{x})$ , so that an efficient law maximizes

$$\mathcal{W}(D_m) = \mathbb{E}(\tilde{x}) - c \int_{D_m \cup (\tau(D_m), \infty)} f(x) dx,$$

which amounts to minimizing the probability of disclosure. Expanding the mandatory disclosure set  $D_m$  will directly increase the disclosure cost for any event that was not disclosed; indirectly, however, expanding  $D_m$  can increase the voluntary disclosure threshold  $\tau(D_m)$  which, then, reduces disclosure costs.

**Lemma 2** *An efficient disclosure law  $D_m^*$  has the form  $(-\infty, \theta^*)$ , where  $\theta^* \in \mathbb{R} \cup \{-\infty\}$ .*

To minimize expected disclosure costs, the law must increase the market posterior expectation conditional on withholding, at a fixed disclosure cost for each event subject to

the law. To do so, it requires the disclosure of the most unfavorable events, which would have been withheld if their disclosure had been voluntary. The efficient law can be fully characterized as a threshold  $\theta^*$  below which events are subject to mandatory disclosure.

Lemma 2 also simplifies the characterization of the voluntary disclosure threshold. The equilibrium voluntary disclosure threshold  $\tau$  makes the marginal voluntary disclosers indifferent between disclosing and not disclosing, satisfying  $\Gamma_1(\tau, \theta) = 0$  where

$$\Gamma_1(t, \theta) = \int_{\theta}^t f(x)x dx - (t - c) \int_{\theta}^t f(x) dx. \quad (2.2)$$

Define the function  $\tau_{\theta}$  as the unique solution to  $\Gamma_1(\tau_{\theta}, \theta) = 0$  for any  $\theta \in [-\infty, \infty)$ .<sup>10</sup> Further, lemma 1 implies that  $\tau_{-\infty}$  is finite so that, by continuity,  $\tau_{\theta}$  is bounded from below by  $\underline{\tau} = \tau_{-\infty} \in \mathbb{R}$ . In other words, the voluntary disclosure threshold can never be lower than  $\underline{\tau}$ . Then, the equilibrium non-disclosure price induced by a mandatory disclosure threshold  $\theta$  is denoted  $P_{\theta}(nd) \equiv \tau_{\theta} - c$  for any  $\theta \in [-\infty, \infty)$ .

The next proposition is our main result and establishes that some events must be subject to mandatory disclosure.

**Proposition 1** *There exists an efficient disclosure law  $D_m^* = (-\infty, \theta^*)$  such that  $\theta^* \in \mathbb{R}$  is uniquely given by*

$$F(\tau_{\theta^*}) - F(\theta^*) - f(\tau_{\theta^*})(\tau_{\theta^*} - \theta^*) = 0. \quad (2.3)$$

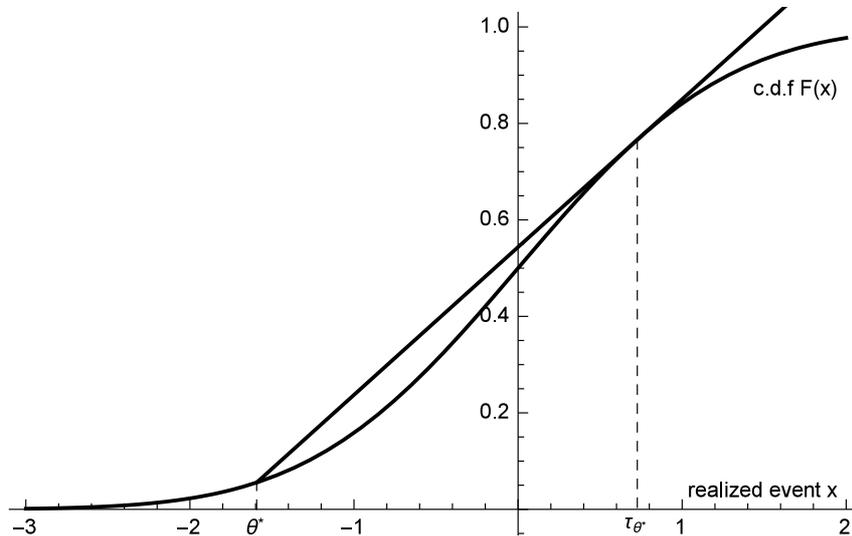
In Figure 1, we plot the solution to the two thresholds for the special case of the Normal distribution. For any  $\theta^*$ , the voluntary disclosure threshold  $\tau_{\theta^*}$  that satisfies (2.3) is obtained at the the point where the line that crosses  $(\theta^*, F(\theta^*))$  is tangent to the c.d.f.. For any placement of  $\theta^*$  (which we know must be set below  $\tau_{\theta^*}$ ), this tangency point is

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<sup>10</sup>Uniqueness follows directly from strict logconcavity and a strictly logconcave distribution truncated from below is also strict logconcave (Bagnoli and Bergstrom 2005).

always above the (mode) mean of the Normal. We generalize this property to any strictly logconcave distribution with mode  $m$  in the next Corollary.

Figure 1: Mandatory and voluntary thresholds with standard Normal and  $c = 1$ .



**Corollary 1** *The mandatory disclosure threshold  $\theta^*$  satisfies  $-\infty < \theta^* < m < \tau_{\theta^*}$ . If the distribution of  $\tilde{x}$  is symmetric, the probability of voluntary disclosure is always greater than the probability of mandatory disclosure.*

The disclosure thresholds  $\theta^*$  and  $\tau_{\theta^*}$  are located at each side of the mode of the distribution. That is, voluntary disclosures are bounded from below by the mode for any non-zero cost. By contrast, absent the efficient mandatory disclosure, the voluntary disclosure threshold becomes arbitrarily small and the voluntary disclosure equilibrium unravels to full disclosure as disclosure cost becomes small (Jovanovic 1982, Verrecchia 1983).

To reconcile this observation to unravelling theory (Viscusi 1978, Milgrom 1981), let us examine the two thresholds as the cost  $c$  becomes small. The unravelling theorems imply that  $\tau_{\theta} - \theta$  converges to zero for any given  $\theta$  as all events that are not subject to the law are voluntarily disclosed. Hence, in equation (2.3),  $F(\tau_{\theta^*}) - F(\theta^*)$  converges to

zero which implies that both  $\tau_{\theta^*}$  and  $\theta^*$  must converge to the mode  $m$ . The equilibrium features unravelling, but via both the voluntary and mandatory disclosure channels.

**Corollary 2** *The optimal mandatory disclosure threshold  $\theta^*$  decreases in cost of disclosure  $c$ , and the voluntary disclosure threshold  $\tau_{\theta^*}$  increases in the cost of disclosure  $c$ .*

We examine next the effect of a change in the variance of the distribution for the special case in which  $F(\cdot)$  is Normal, extending the comparative static of Verrecchia (1990) in the case of an efficient mandatory disclosure law.<sup>11</sup>

**Corollary 3** *Suppose that  $\tilde{x} \sim N(\mu, \sigma^2)$ , both unconditional probability of mandatory and voluntary disclosure increase in the variance  $\sigma^2$ .*

In the context of normally-distributed random variables, both  $\tilde{x}$  and  $c$  can be standardized by dividing by the standard deviation  $\sigma$  so that an increase in  $\sigma$  is equivalent to a reduction in (standardized) disclosure cost. Therefore, from Corollary 2, both the probability of voluntary disclosure and the probability of mandatory disclosure must increase.

### 3. Further analyses

#### 3.1. Bounded support

If  $\tilde{x}$  has a support  $[\underline{x}, \infty)$  with  $\underline{x} > -\infty$  bounded from below, it is possible that no event may be sufficiently unfavorable to be subject to mandatory disclosure, and thus the efficient disclosure law would prescribe a zero probability of mandatory disclosure.

**Proposition 2** *The mandatory disclosure threshold  $\theta^*$  satisfies  $\theta^* > \underline{x}$  and is uniquely given by equation (2.3), if and only if  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) < 0$ .*

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<sup>11</sup>Verrecchia (1990) interprets an increase in the variance of the private signal as greater information quality possessed by the seller, because the seller features a greater dispersion in posterior expectations (Ganuza and Penalva 2010).

Because the  $\tau_\theta$  is decreasing in  $c$ , we can restate the condition for a non-zero probability of mandatory disclosure as an upper bound  $c < \bar{c}$ , where  $\bar{c}$  is the solution to  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) = 0$ , if one exists, or  $\bar{c} = 0$  otherwise. That is, mandatory disclosure is typically desirable provided the disclosure cost is sufficiently small. There are also classes of bounded distributions for which no mandatory disclosure is desirable regardless of the disclosure cost (i.e.,  $\bar{c} = 0$ ). Since the efficient disclosure threshold must lie below the mode, any distribution with decreasing density (e.g., an exponential distribution or some Beta distributions) must be that such that  $\theta^* = \underline{x}$  and the efficient law features a zero probability of mandatory disclosure.

A support of the form  $(-\infty, \bar{x})$  bounded from above exhibits similar properties. As long as the solution to equation (2.3) exhibits some voluntary disclosure, that is,  $\tau_{\theta^*} < \bar{x}$ , this solution remains the efficient mandatory disclosure threshold.<sup>12</sup> Otherwise, the efficient induced voluntary disclosure threshold should be  $\tau_{\theta^*} = \bar{x}$ . This case is a mirror image to the previous discussion to the extent that the probability of mandatory disclosure will be non-zero but the probability of voluntary disclosure will be zero. In this case, mandatory disclosure may crowd out voluntary disclosure completely.

### 3.2. Increasing disclosure costs

In this section, we extend the main result if the cost of disclosure increases with respect to the content of the information. For example, damages and defects may be easier to verify than potential benefits or, in the context of financial numbers, current losses of value may be easier to observe (in current cash flows) than unrealized future gains.

We represent the cost as a continuously differentiable, strictly positive and bounded function  $c(x) > 0$ . To ensure that the voluntary disclosure set is always an upper interval, we assume that  $x - c(x)$  is an increasing function, i.e.,  $0 \leq c'(x) < 1$ .

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<sup>12</sup>Recall  $\tau_\theta$  is given by a solution to equation 2.2 and, even if the support is bounded, this function need not satisfy  $\tau_\theta < \bar{x}$  (of course, a solution implies that voluntary disclosure has probability zero).

**Proposition 3** *If the disclosure cost is a continuously differentiable and bounded function  $c(x) > 0$ , where  $0 \leq c'(x) < 1$ , there exists an efficient disclosure law  $D_m^* = (-\infty, \theta^*)$  such that  $\theta^* \in \mathbb{R}$ .*

This result is intuitive as an increasing cost implies that the relative cost of the voluntary disclosures at the upper tail of the distribution are large, especially relative to the (lower) cost of mandatory disclosures on the lower tail. Hence, increasing cost work to increase the desirability of mandatory disclosure of unfavorable events.

### 3.3. Social value of information

In the baseline model, we assume that the price is a linear function of the rational expectation of the asset value. We generalize this form to  $P(x) = \phi(x)$  when a disclosure is made and, when possible,  $P(nd) = \phi(\mathbb{E}(\tilde{x}|r(\tilde{x}) = nd))$ , where  $\phi(\cdot)$  is strictly increasing and twice-continuously-differentiable.

In the special case of  $\phi(\cdot)$  being linear, the model is pure-exchange to the extent that information does not affect the expected selling price. If  $\phi(\cdot)$  is strictly convex, the expected selling price is always greater in the presence of more precise information such as, for example, price-protected buyers making additional decisions conditional on the public information. Vice-versa, as long as we write payoffs in seller utility, a strictly concave  $\phi(\cdot)$  corresponds to a risk-averse seller who prefers less informative public reports.<sup>13</sup> Note that we no longer require logconcavity in this section so, if we set  $\phi(\cdot)$  linear, this section demonstrates that our main result holds even if the distribution is no logconcave.

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<sup>13</sup>We give, below, two more formal examples. First, suppose that buyers are fully price-protected (i.e., the price is their expected surplus) and realize a cash flow  $\max_I \tilde{x}I - I^2/2$ , where  $I$  is an investment to be made after buying. The optimal investment based on their information is  $I^* = \mathbb{E}(\tilde{x}|r)$  which yields a buying price  $P(r) = \mathbb{E}(\tilde{x}|r)^2/2$  implying a convex payoff function  $\phi(x) = x^2/2$ . More generally, a convex payoff function is commonly-used in this type of model to model social benefits of information (Kamenica and Gentzkow 2011, Frankel and Kartik 2014). Second, suppose that the seller has a utility function  $u(\mathbb{E}(\tilde{x}|r)) - c\mathbf{1}(r \neq nd)$  where  $u(\cdot)$  is a strictly concave utility function. As is usual with risk-averse players (Spear and Srivastava 1987), one can rewrite payoffs in utilities, setting  $\phi(\cdot) = u(\cdot)$ . This slightly deviates from Verrecchia (1983), in which risk-aversion is modelled as mean-variance optimization but is consistent with the approach developed in Jorgensen and Kirschenheiter (2015).

Define  $x_{nd} = \mathbb{E}(\tilde{x}|\tilde{x} \leq \tau, \tilde{x} \notin D_m)$ . The equilibrium  $\mathcal{E}(D_m)$ , if it exists, features a threshold  $\tau$  such that  $x$  would be voluntarily disclosed if and only if  $x > \tau$  where  $\tau$  is defined as the indifference point between disclosing voluntarily and not disclosing:

$$\phi(\tau) - c - \phi(x_{nd}) = 0.^{14} \quad (3.1)$$

To characterize an efficient disclosure law, we borrow a formulation of the disclosure problem from Bertomeu and Cheynel (2015) by replacing the mandatory reporting set  $D_m$  with an indicator function  $\Theta(x) \in \{0, 1\}$  where  $\Theta(x) = 1$  if and only if  $x \notin D_m$ . Thus, there exists a unique  $D_m$  for each function  $\Theta(\cdot)$  and, vice-versa, so we may solve for either the set or the indicator function equivalently.

An efficient disclosure law is characterized by a solution  $(\Theta^*(\cdot), x_{nd}^*, \tau^*)$  to the following constrained optimization program:

$$(Q) \quad \max_{\Theta(\cdot), x_{nd} \in \mathbb{R}, \tau \in \mathbb{R}} \int_{-\infty}^{\tau} (\phi(x_{nd})\Theta(x) + (\phi(x) - c)(1 - \Theta(x)))f(x)dx + \int_{\tau}^{+\infty} (\phi(x) - c)f(x)dx$$

subject to (3.1) and

$$x_{nd} \int_{-\infty}^{\tau} \Theta(x)f(x)dx - \int_{-\infty}^{\tau} x\Theta(x)f(x)dx = 0. \quad (3.2)$$

Constraint (3.2) states that  $x_{nd} = \mathbb{E}(\tilde{x}|\tilde{x} \leq \tau, \Theta(\tilde{x}) = 1)$  must be consistent with Bayes rule and corresponds to (ii) in Definition 1. Within this formulation, the choice of an efficient disclosure law is linear in  $\Theta(\cdot)$ . As we prove next, this property can be used to show that the problem has a bang-bang solution such that the mandatory disclosure takes the form of a threshold.

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<sup>14</sup>Of particular note is that if the distribution has a left-tail which is not sub-exponential, the above equation may not admit a solution to  $\tau$  in  $\mathbb{R}$ . However, our problem enjoys an additional degree of freedom in choosing  $D_m$ , which can truncate a tiny portion of the left-tail to rule out the non-existence problem. So we do not have to impose a sub-exponential left-tail assumption here. What we show in the following is that truncating the left-trail does not only serve to ensure the existence of voluntary disclosure equilibrium, when there exists not otherwise, it is also more efficient than any other laws that entails equilibria.

**Proposition 4** *If the pricing function  $\phi(\cdot)$  is strictly convex, an efficient disclosure law must be such that all sufficiently unfavorable information is subject to mandatory disclosure, that is,  $D_m^* = (-\infty, \theta^*)$ , where  $-\infty < \theta^* < \tau^* < -\infty$ .*

When the pricing function  $\phi(\cdot)$  is strictly convex, the intuition of the prediction can be seen from Jensen's inequality. From ex-ante perspective, the seller prefers more volatility in price because the expectation of a convex function is greater than the convex function of an expectation. If voluntary disclosure is not possible and the verification cost is sufficiently small, full disclosure is always preferred to no disclosure at all.

However, when the choice space is larger than full disclosure or no disclosure, it is optimal to mandate disclosing only non-mediocre news. No matter how small the verification cost is, the optimal disclosure is never full disclosure. The reason is that the volatility is generated with a cost. The mediocre news does not create adequate volatility to justify the cost. Thus the non-disclosure set will only shrink to a point where the marginal increase in volatility equals the marginal cost of disclosure.

When voluntary disclosure is considered, the efficient disclosure law can free-ride on the incentive of seller to disclose favorable information voluntarily and only mandates disclosing unfavorable information. Disclosing unfavorable news can benefit mediocre sellers who do not have to disclose.

We examine next the case of concave pricing, and find that, in contrast to the previous analysis, the efficient disclosure law need not focus on unfavorable news.

**Proposition 5** *If the pricing function  $\phi(\cdot)$  is strictly concave, an efficient disclosure law may not mandate disclosing some unfavorable information in  $(-\infty, +\bar{\kappa})$ , where  $\bar{\kappa} \in (-\infty, \tau^*)$ .*

When the seller is risk-averse, disclosing increases volatility in price, which causes a secondary cost of disclosure above and beyond the disclosure cost. The problem of mandating disclosure of bad news is that it induces a disclosure equilibrium in which only

extreme news are disclosed, and thus raise the price risk perceived by a risk-averse seller. To gain further intuition, consider the case of infinite risk-aversion in which the seller ex-ante considers the minimum price that could be realized. This extreme preference will cause the seller to prefer the maximal possible lower tail non-disclosure region and, therefore, would prefer no mandatory disclosure. More generally, risk-aversion leads the seller to prefer withholding bad news.

## 4. Conclusion

In this paper, we examine whether voluntary disclosure, which is typically analyzed as a market solution to disclosure problems, carries externalities that cause a demand for regulation. We view here the social value of regulation as an ex-ante social pre-commitment prior to information being known, which would not be voluntarily adopted by firms if they cannot contract in a state of ignorance. A suitably designed regulation that mandates the disclosure of bad news mitigates the social cost of excessive voluntary disclosure.

The model has several new implications. First, we argue that tests of market responses must condition the hypothesis on whether the news is voluntary (i.e., its omission would not cause a direct cost) or mandatory, and note that mandatory disclosure should be expected to convey negative news if they are optimally designed. Second, we develop several results that are in sharp contrast with models of pure voluntary disclosure, such that (i) market reactions to withholding need not be large when disclosure costs are low (since only near-modal events are disclosed), (ii) all voluntary disclosures should imply positive market reactions (rather than in expectation) and (iii) the most likely events should usually be withheld. Taken together, these predictions also offer new empirical strategies to test whether disclosure laws appear to be efficient.

## 5. Appendix

**Proof of Lemma 1:** The proof follows from the intermediate value theorem, letting  $\underline{t} = \inf \mathbb{R} \setminus D_m$ : (a)  $t - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, x \leq t)$  is strictly positive for  $t > \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m) + c$ ,<sup>15</sup> and (b)  $\lim_{t \rightarrow \underline{t}} t - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t) = -c$  if  $\underline{t}$  is finite or  $\lim_{t \rightarrow \underline{t}} t - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t) = -\infty$  if  $\underline{t} = -\infty$  from the fact that  $f(\cdot)$  has sub-exponential tail. Hence, there exists at least one solution to  $t - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t) = 0$  and all solutions are in  $(\underline{t}, \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m) + c)$ , that is,  $\sup_{\mathcal{E}} \tau(D_m|\mathcal{E}) \in \mathbb{R}$ . To conclude the proof, note that the function  $t - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t)$  is continuous in  $t$ , so that for any convergent sequence  $\{t_i\}$  that satisfies  $t_i - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t_i) = 0$  for all  $i$ , its limit  $t_\infty$  satisfies  $t_\infty - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t_\infty) = 0$  and, therefore,  $t_\infty = \max_{\mathcal{E}} \tau(D_m|\mathcal{E})$ .  $\square$

**Proof of Lemma 2:** Let  $D_m^*$  be an efficient disclosure law with  $\tau \equiv \tau(D_m^*)$  and  $\lambda \equiv \{x : x > \tau\}$ . Suppose by contradiction that  $(a_1, a_2) \subset D_m^*$  where  $-\infty < a_1 < a_2 \leq \tau$ . Below, we construct an alternative disclosure set with a lower probability of disclosure.

Define  $\epsilon_1, \epsilon_2 > 0$  such that (i)  $a_1 + \epsilon_2 < a_2$  and (ii)  $F(a_1 + \epsilon_2) - F(a_1) = F(a_1) - F(a_1 - \epsilon_1)$  and an alternative mandatory disclosure set  $D'_m$  by  $D'_m = D_m^* \cup (a_1 - \epsilon_1, a_1) \setminus (a_1, a_1 + \epsilon_2)$ . Then, let  $\tau' \equiv \tau(D'_m)$  and  $\lambda' \equiv (\tau', \infty)$ . By construction,  $t - c - \mathbb{E}(\tilde{x}|\tilde{x} \leq t, \tilde{x} \notin D'_m) < t - c - \mathbb{E}(\tilde{x}|\tilde{x} \leq t, \tilde{x} \notin D_m)$  for any  $t$ , which implies that  $\tau' > \tau$ . It then follows that

$$\int_{x \in D'_m \cup \lambda'} f(x) dx = \int_{x \in D_m^* \cup \lambda'} f(x) dx < \int_{x \in D_m^* \cup \lambda} f(x) dx.$$

Therefore, the set  $D'_m$  would feature lower expected disclosure costs than  $D_m^*$ , a contradiction.  $\square$

**Proof of Proposition 1:** Strict logconcavity of  $f(x)$  implies that  $t - \mathbb{E}(\tilde{x}|\theta \leq x \leq t)$

<sup>15</sup>This is from the fact that  $\mathbb{E}[\tilde{x}] \in \mathbb{R}$  implies that  $\mathbb{E}(\tilde{x}|\tilde{x} \notin D_m) \in \mathbb{R}$ .

is strictly increasing in  $t$  (Prékopa 1973, Bagnoli and Bergstrom 2005). Hence,

$$\begin{aligned} \frac{\partial(t - \mathbb{E}(\tilde{x}|\theta \leq x \leq t))}{\partial t} \Big|_{t=\tau_\theta} &= 1 - \frac{f(\tau_\theta)\tau_\theta \int_\theta^{\tau_\theta} f(x)dx - f(\tau_\theta) \int_\theta^{\tau_\theta} f(x)x dx}{\left(\int_\theta^{\tau_\theta} f(x)dx\right)^2} \\ &= 1 - \frac{f(\tau_\theta)}{\int_\theta^{\tau_\theta} f(x)dx} \underbrace{\left(\tau_\theta - \frac{\int_\theta^{\tau_\theta} f(x)x dx}{\int_\theta^{\tau_\theta} f(x)dx}\right)}_{=c \text{ (from (2.2))}} > 0 \end{aligned}$$

which can be rewritten as  $F(\tau_\theta) - F(\theta) - cf(\tau_\theta) > 0$ .

It then follows from the implicit function theorem on  $\Gamma_1(\tau_\theta, \theta) = 0$ ,

$$\begin{aligned} \frac{\partial\tau_\theta}{\partial\theta} &= -\frac{\partial\Gamma_1(t, \theta)}{\partial\theta} / \frac{\partial\Gamma_1(t, \theta)}{\partial t} \Big|_{t=\tau_\theta} \\ &= \frac{(\tau_\theta - \theta - c)f(\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} > 0 \end{aligned} \tag{5.1}$$

because  $\tau_\theta - c = P_\theta(nd) > \theta$ . This implies that  $\underline{\tau} = \tau_{-\infty}$ .

Let  $\mathcal{C}(\theta)$  denote the expected disclosure cost as a function of  $\theta$ ,

$$\begin{aligned} \mathcal{C}(\theta) &= F(\theta)c + (1 - F(\tau_\theta))c, \\ \mathcal{C}'(\theta) &= \left(f(\tau_\theta) - \frac{\partial\tau_\theta}{\partial\theta}f(\tau_\theta)\right)c \\ &= f(\theta)c \left(\frac{F(\tau_\theta) - F(\theta) - (\tau_\theta - \theta)f(\tau_\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)}\right). \end{aligned} \tag{5.2}$$

Because the denominator in the right-hand side of (5.2) is positive, the sign of  $\mathcal{C}'(\theta)$  is equal to the sign of  $F(\tau_\theta) - F(\theta) - (\tau_\theta - \theta)f(\tau_\theta)$  which, since  $\tau_\theta$  is bounded, is strictly negative for  $\theta$  sufficiently small. Hence,  $\theta^* = -\infty$  is not efficient, so that  $\theta^* \in \mathbb{R}$ .

By equation (5.2), the mandatory disclosure threshold  $\theta^*$  must satisfy the following necessary first-order condition

$$F(\tau_{\theta^*}) - F(\theta^*) - (\tau_{\theta^*} - \theta^*)f(\tau_{\theta^*}) = 0. \tag{5.3}$$

From the mean value theorem, there exists  $y^* \in (\theta^*, \tau_{\theta^*})$  such that:

$$f(y^*) = \frac{F(\tau_{\theta^*}) - F(\theta^*)}{\tau_{\theta^*} - \theta^*} = f(\tau_{\theta^*}).$$

By strict logconcavity, the distribution is single-peaked, so that  $y^*$  and  $\tau_{\theta^*}$  must be placed on each side of the mode  $m$ , i.e.,  $y^* < m < \tau_{\theta^*}$ , which implies:  $\theta^* < m < \tau_{\theta^*}$  and

$$f(\theta^*) < f(\tau_{\theta^*}). \quad (5.4)$$

We prove next that the solution to equation (5.3) is unique. For any  $\theta < m$ , define  $\tau_2(\theta)$  as a solution to  $\Gamma_2(t, \theta) = 0$  with  $\theta < m < t$  and  $f(\theta) < f(t)$  where

$$\Gamma_2(t, \theta) = F(t) - F(\theta) - (t - \theta)f(t).$$

Note that (i)  $F(m) - F(\theta) - (m - \theta)f(m) < 0$  because  $F(m) - F(\theta) \equiv \int_{\theta}^m f(x)dx < \int_{\theta}^m f(m)dx \equiv (m - \theta)f(m)$ , and  $\lim_{t \rightarrow \infty} \Gamma(t, \theta) = \lim_{t \rightarrow \infty} F(t) - F(\theta) - \lim_{t \rightarrow \infty} tf(t) + \lim_{t \rightarrow \infty} \theta f(t) = 1 - F(\theta) > 0$  because  $\lim_{t \rightarrow \infty} tf(t) = 0$  as implied by a finite mean, and (ii)  $\partial \Gamma_2 / \partial t = -(t - \theta)f'(t)$  is strictly positive for  $t > m$ . Hence,  $\tau_2(\theta)$  is unique and is a function from  $(-\infty, m)$  to  $(m, \infty)$ . Further, the voluntary threshold  $\tau_{\theta^*}$  must satisfy the cost-minimizing optimality condition  $\tau_2(\theta^*)$ , which implies that  $\tau_2(\theta^*) = \tau_{\theta^*}$ . We know from (5.1) that  $\tau_{\theta}$  is increasing in  $\theta$  and, applying the implicit function theorem to  $\Gamma_2(t, \theta)$ ,

$$\tau_2'(\theta) = \frac{(\tau_2(\theta) - \theta)f'(\tau_2(\theta))}{f(\tau_2(\theta)) - f(\theta)} < 0, \text{ for } \tau_2(\theta) > m > \theta \text{ and } f(\tau_2(\theta)) > f(\theta).$$

This implies that  $\tau_{\theta}$  and  $\tau_2(\theta)$  cross once and, hence, the solution to  $\tau_{\theta^*} = \tau_2(\theta^*)$  is unique.  $\square$

**Proof of Corollary 1:** The fact that  $-\infty < \theta^* < m < \tau_{\theta^*}$  is shown in the proof of proposition 1. For the special case in which the distribution of  $\tilde{x}$  is symmetric,  $\tau_{\theta^*} - m = m - y^*$  so that  $Prob(\tilde{x} < y^*) = Prob(\tilde{x} > \tau_{\theta^*})$ , implying that the probability of mandatory disclosure  $Prob(\tilde{x} < \theta^*) < Prob(\tilde{x} < y^*)$  is lower than the probability of voluntary disclosure.  $\square$

**Proof of Corollary 2:** Applying implicit function theorem to  $\Gamma_1(\tau_{\theta^*}, \theta^*) = 0$  and  $\Gamma_2(\tau_{\theta^*}, \theta^*) = 0$ :

$$\begin{aligned} \begin{pmatrix} \frac{\partial \theta^*}{\partial c} \\ \frac{\partial \tau_{\theta^*}}{\partial c} \end{pmatrix} &= - \begin{pmatrix} \frac{\partial \Gamma_1(t, \theta)}{\partial \theta} \Big|_{\theta=\theta^*, t=\tau_{\theta^*}} & \frac{\partial \Gamma_1(t, \theta)}{\partial t} \Big|_{\theta=\theta^*, t=\tau_{\theta^*}} \\ \frac{\partial \Gamma_2(t, \theta)}{\partial \theta} \Big|_{\theta=\theta^*, t=\tau_{\theta^*}} & \frac{\partial \Gamma_2(t, \theta)}{\partial t} \Big|_{\theta=\theta^*, t=\tau_{\theta^*}} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial \Gamma_1(t, \theta)}{\partial c} \Big|_{\theta=\theta^*, t=\tau_{\theta^*}} \\ \frac{\partial \Gamma_2(t, \theta)}{\partial c} \Big|_{\theta=\theta^*, t=\tau_{\theta^*}} \end{pmatrix} \\ &= - \begin{pmatrix} (\tau_{\theta^*} - \theta^* - c)f(\theta^*) & cf(\tau_{\theta^*}) - (F(\tau_{\theta^*}) - F(\theta^*)) \\ -f(\theta^*) + f(\tau_{\theta^*}) & -(\tau_{\theta^*} - \theta^*)f'(\tau_{\theta^*}) \end{pmatrix}^{-1} \begin{pmatrix} \Delta \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \frac{F(\tau_{\theta^*}) - F(\theta^*)}{Z} (\tau_{\theta^*} - \theta^*) f'(\tau_{\theta^*}) \\ \frac{F(\tau_{\theta^*}) - F(\theta^*)}{Z} (f(\tau_{\theta^*}) - f(\theta^*)) \end{pmatrix}, \end{aligned}$$

where  $\Delta = F(\tau_{\theta^*}) - F(\theta^*)$  and

$$Z = (f(\tau_{\theta^*}) - f(\theta^*))(F(\tau_{\theta^*}) - F(\theta^*) - cf(\tau_{\theta^*})) + (\tau_{\theta^*} - \theta^*)(c - \tau_{\theta^*} + \theta^*)f(\theta^*)f'(\tau_{\theta^*}). \quad (5.5)$$

We know that (i)  $f'(\tau_{\theta^*}) < 0$  from corollary 1 and (ii)  $f(\tau_{\theta^*}) - f(\theta^*) > 0$  from (5.4) so that  $Sign(\frac{\partial \tau_{\theta^*}}{\partial c}) = -Sign(\frac{\partial \theta^*}{\partial c}) = Sign(Z)$ , and  $Z > 0$  is implied by (iii)  $\tau_{\theta^*} - c > \theta^*$  from (2.1), and (iv)  $F(\tau_{\theta^*}) - F(\theta^*) - cf(\tau_{\theta^*}) > 0$  from (iii) and (2.3).  $\square$

**Proof of Corollary 3:** Without loss of generality, we set  $\mu = 0$ . Denote the p.d.f. and c.d.f. of the standard normal distribution as  $\phi(\cdot)$  and  $\Phi(\cdot)$  respectively. The probability of mandatory and voluntary disclosure can be written respectively as:

$$\begin{aligned} \text{Prob}(x < \theta^*) &= \Phi\left(\frac{\theta^*}{\sigma}\right), \\ \text{Prob}(x > \tau_{\theta^*}) &= 1 - \Phi\left(\frac{\tau_{\theta^*}}{\sigma}\right). \end{aligned}$$

We do the following change of variable to  $\Gamma_1(\tau_{\theta^*}, \theta^*) = 0$  and  $\Gamma_2(\tau_{\theta^*}, \theta^*) = 0$ :

$$\int_{\theta^*/\sigma}^{\tau_{\theta^*}/\sigma} \phi(u) u \, du - \left(\frac{\tau_{\theta^*}}{\sigma} - \frac{c}{\sigma}\right) \int_{\theta^*/\sigma}^{\tau_{\theta^*}/\sigma} \phi(u) \, du = 0,$$

$$\Phi\left(\frac{\tau_{\theta^*}}{\sigma}\right) - \Phi\left(\frac{\theta^*}{\sigma}\right) - \left(\frac{\tau_{\theta^*}}{\sigma} - \frac{\theta^*}{\sigma}\right) \phi\left(\frac{\tau_{\theta^*}}{\sigma}\right) = 0.$$

From the proof of corollary 2, we have that  $\frac{\theta^*}{\sigma}$  decreases in  $\frac{c}{\sigma}$  and  $\frac{\tau_{\theta^*}}{\sigma}$  increases in  $\frac{c}{\sigma}$ . Hence, it proves that  $\frac{\theta^*}{\sigma}$  increases in  $\sigma$  and  $\frac{\tau_{\theta^*}}{\sigma}$  decreases in  $\sigma$ .  $\square$

**Proof of Proposition 2:** We prove the **if** and **only** part separately.

**If:** As shown in the proof of Proposition 1, the sign of the F.O.C. for minimizing the expected disclosure cost when evaluated at  $\theta = \underline{x}$  is determined by the sign of  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}})$ . The set  $\{(F(\cdot), c) | F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) < 0\}$  is not empty, as shown below.

Assuming  $\underline{x} < m < +\infty$ ,  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) < 0$  if  $\underline{x} < \tau_{\underline{x}} \leq m$ . If  $\tau_{\underline{x}} > m$ ,  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}})$  is increasing in  $\tau_{\underline{x}}$ , i.e.,  $\frac{\partial(F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}))}{\partial \tau_{\underline{x}}} = -(\tau_{\underline{x}} - \underline{x})f'(\tau_{\underline{x}}) > 0$ . Also  $\lim_{\tau_{\underline{x}} \rightarrow +\infty} F(\tau_{\underline{x}}) - F(\underline{x}) + (\underline{x} - \tau_{\underline{x}})f(\tau_{\underline{x}}) = F(+\infty) - F(\underline{x}) + (\underline{x} - \infty)f(+\infty) = 1 - F(\underline{x}) > 0$ . By intermediate value theorem, there exists a unique  $\hat{\tau}_{\underline{x}} > m$  such that  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) \geq (\leq) 0$  if  $\tau_{\underline{x}} \geq (\leq) \hat{\tau}_{\underline{x}}$ . Since  $\tau_{\underline{x}}$  increases in  $c$ , there exists a unique  $\bar{c} > 0$  such that  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) < 0$  if (and only if)  $c < \bar{c}$ .

$F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) < 0$  implies that  $\underline{x} < m < +\infty$ . The proof of uniqueness in Proposition 1 goes through here when the support is replaced by  $[\underline{x}, +\infty)$ .

**Only if:**  $F(\tau_{\underline{x}}) - F(\underline{x}) - (\tau_{\underline{x}} - \underline{x})f(\tau_{\underline{x}}) \geq 0$  would contradict the fact there exists a unique solution to equation (2.3) for  $x \in [\underline{x}, +\infty)$ .  $\square$

**Proof of Proposition 3:** It can be easily checked that lemma 1 holds because  $c(x)$  is bounded. Lemma 2 holds for increasing cost function  $c(x)$  as well, for the reason that the constructed alternative law  $D'_m$  features the same probability of mandatory disclosure as  $D_m$ , while disclosing  $(a_1 - \epsilon_1, a_1)$  is less costly than disclosing  $(a_1, a_1 + \epsilon_2)$ .

As in the proof of Proposition 1, differentiating the function  $t - \mathbb{E}(\tilde{x}|\theta \leq x \leq t)$ , which is strictly increasing, we know that  $F(\tau_\theta) - F(\theta) - c(\tau_\theta)f(\tau_\theta) > 0$ .

Applying the implicit function theorem to  $\Gamma(\tau_\theta, \theta) = 0$ , i.e.,

$$\begin{aligned} \frac{\partial \tau_\theta}{\partial \theta} &= -\frac{\partial \Gamma(t, \theta)}{\partial \theta} / \frac{\partial \Gamma(t, \theta)}{\partial t} \Big|_{t=\tau_\theta} \\ &= \frac{(\tau_\theta - \theta - c(\tau_\theta))f(\theta)}{F(\tau_\theta) - F(\theta) - c(\tau_\theta)f(\tau_\theta)} > 0. \end{aligned}$$

From step 2,  $F(\tau_\theta) - F(\theta) - c(x)f(\tau_\theta) > 0$ . Also note that  $\tau_\theta - c(\tau_\theta) = P_\theta(nd) > \theta$ .

Let  $\mathcal{C}(\theta)$  denote the expected disclosure cost as a function of  $\theta$ , that is,

$$\begin{aligned}
\mathcal{C}(\theta) &= \int_{-\infty}^{\theta} f(x)c(x)dx + \int_{\tau_{\theta}}^{+\infty} f(x)c(x)dx. \\
\mathcal{C}'(\theta) &= f(\theta)c(\theta) - f(\tau_{\theta})c(\tau_{\theta})\frac{\partial\tau_{\theta}}{\partial\theta} \\
&= f(\theta)c(\theta) - f(\tau_{\theta})c(\tau_{\theta})\left(\frac{(\tau_{\theta} - \theta - c(\tau_{\theta}))f(\theta)}{F(\tau_{\theta}) - F(\theta) - c(\tau_{\theta})f(\tau_{\theta})}\right) \\
&= f(\theta)\left(c(\theta) - f(\tau_{\theta})c(\tau_{\theta})\left(\frac{\tau_{\theta} - \theta - c(\tau_{\theta})}{F(\tau_{\theta}) - F(\theta) - c(\tau_{\theta})f(\tau_{\theta})}\right)\right) \\
&< f(\theta)\left(c(\tau_{\theta}) - f(\tau_{\theta})c(\tau_{\theta})\left(\frac{\tau_{\theta} - \theta - c(\tau_{\theta})}{F(\tau_{\theta}) - F(\theta) - c(\tau_{\theta})f(\tau_{\theta})}\right)\right) \\
&= f(\theta)c(\tau_{\theta})\left(\frac{F(\tau_{\theta}) - F(\theta) - (\tau_{\theta} - \theta)f(\tau_{\theta})}{F(\tau_{\theta}) - F(\theta) - c(\tau_{\theta})f(\tau_{\theta})}\right).
\end{aligned}$$

It has been shown in the proof of proposition 1 that  $F(\tau_{\theta}) - F(\theta) - (\tau_{\theta} - \theta)f(\tau_{\theta})$  is strictly negative for sufficient small  $\theta$ . Hence  $\mathcal{C}'(\theta)$  is strictly negative for sufficiently small  $\theta$ , i.e.,  $\theta = -\infty$  is not optimal.  $\square$

#### Proof of Proposition 4:

We prove this result in several steps.

**Step 1:** We argue that  $\tau^* = \max\{\tau : \phi(\tau) - c - \phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau, \Theta^*(\tilde{x}) = 1)) = 0\}$ , i.e.,  $\tau^*$  is the maximal threshold that can be sustained in equilibrium. To see this, note that, for any two solutions  $\tau' > \tau''$  to  $\phi(\tau) - c - \phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau, \Theta^*(\tilde{x}) = 1)) = 0$ , it holds that (i)  $\phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau'', \Theta^*(\tilde{x}) = 1)) > \phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau', \Theta^*(\tilde{x}) = 1))$ , and (ii)  $\phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau'', \Theta^*(\tilde{x}) = 1)) > \phi(x) - c$  for any  $x \in (\tau'', \tau')$ . This implies that the objective function is greater under  $\tau''$  than under  $\tau'$ .

**Step 2:** In this step, we show that  $\Theta^*(x)$  cannot be discontinuous at  $\tau^*$ . According to definition 2 (ii), we can always set  $\Theta(x) = 1$  for  $x \geq \tau$ . By contradiction, suppose that  $\Theta^*(x) = 0$  for  $x \in (\tau^* - \varepsilon, \tau^*)$  for some  $\varepsilon > 0$ . Set  $\Theta^{**}(x) \equiv \Theta^*(x)$  except that  $\Theta^{**}(x) = 1$  for  $x \in (\tau^* - \varepsilon, \tau^*)$ .  $\Theta^{**}(x)$  is more efficient than  $\Theta^*(x)$  because; (i)  $\tau^{**} > \tau^*$ , for the

reason that

$$\phi(\tau^*) - c - \phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau^*, \Theta^{**}(\tilde{x}) = 1)) < 0,$$

(ii)  $\phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau^{**}, \Theta^{**}(\tilde{x}) = 1)) > \phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau^*, \Theta^*(\tilde{x}) = 1))$ , (iii) for any  $x \in (\tau^* - \varepsilon, \tau^{**}]$ ,  $\phi(\mathbb{E}(\tilde{x}|\tilde{x} \leq \tau^{**}, \Theta^{**}(\tilde{x}) = 1)) \geq \phi(x) - c$ .

**Step 3:** We show a necessary property of  $\tau^*$ :

From step 1, which shows that  $\tau^*$  is the maximal solution to the constraint (3.1) at optimality, we know that:

$$\frac{\partial \left( \phi(\tau) - c - \phi \left( \frac{\int_{-\infty}^{\tau} \Theta^*(x) f(x) dx}{\int_{-\infty}^{\tau} \Theta^*(x) f(x) dx} \right) \right)}{\partial \tau} \Big|_{\tau=\tau^*} \geq 0,$$

which is well-defined from step 2 (continuity implies differentiability of the integral).

Hence

$$\begin{aligned} & \frac{\partial \left( \phi(\tau) - c - \phi \left( \frac{\int_{-\infty}^{\tau} \Theta^*(x) f(x) dx}{\int_{-\infty}^{\tau} \Theta^*(x) f(x) dx} \right) \right)}{\partial \tau} \Big|_{\tau=\tau^*} \\ &= \phi'(\tau^*) - \phi'(x_{nd}^*) \left( \frac{\Theta^*(\tau^*) f(\tau^*)}{\int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx} \right) (\tau^* - x_{nd}^*) \\ &= \frac{\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \phi'(x_{nd}^*) f(\tau^*) (\tau^* - x_{nd}^*)}{\int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx} \geq 0, \end{aligned}$$

which implies that  $\tau^*$  must satisfy:

$$\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \phi'(x_{nd}^*) f(\tau^*) (\tau^* - x_{nd}^*) \geq 0. \quad (5.6)$$

**Step 4:** Taking the F.O.C.s of the Lagrangian, we get:

$$\begin{aligned}\frac{\partial L}{\partial \Theta(x)} &= f(x) (\phi(x_{nd}^*) - \phi(x) + \gamma_2(x_{nd}^* - x) + c), \text{ for each } x, \\ \frac{\partial L}{\partial \tau} \Big|_{\tau=\tau^*} &= \gamma_1 \phi'(\tau^*) + \gamma_2 f(\tau^*) (x_{nd}^* - \tau^*) = 0, \\ \frac{\partial L}{\partial x_{nd}} \Big|_{x_{nd}=x_{nd}^*} &= \phi'(x_{nd}^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \gamma_1 \phi'(x_{nd}^*) + \gamma_2 \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx = 0.\end{aligned}$$

Define

$$S \equiv \frac{\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx}{\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \phi'(x_{nd}^*) f(\tau^*) (\tau^* - x_{nd}^*)}.$$

Note that  $S > 1$  from inequality (5.6).

Solving for  $\gamma_2$ , we get:

$$\gamma_2 = -S \phi'(x_{nd}^*).$$

Hence, the sign of  $\frac{\partial L}{\partial \Theta(x)}$  depends on the sign of

$$\Sigma(x) \equiv \phi(x_{nd}^*) - \phi(x) - S \phi'(x_{nd}^*) (x_{nd}^* - x) + c. \quad (5.7)$$

$\phi(\cdot)$  is strictly convex, i.e.,  $\phi''(x) > 0$ , which implies that:

$$\begin{aligned}\frac{\partial \Sigma(x)}{\partial x} &= S \phi'(x_{nd}^*) - \phi'(x) > 0 \text{ for } x < x_{nd}^*, \text{ (from } S > 1 \text{ and } \phi''(\cdot) > 0), \\ \frac{\partial^2 \Sigma(x)}{\partial x^2} &= -\phi''(x) < 0.\end{aligned}$$

Hence, in this case,  $\Sigma(x)$  is strictly concave and increasing for  $x < x_{nd}^* \in \mathbb{R}$ , with  $\Sigma(x_{nd}^*) = c > 0$ , which implies that there exists a  $\theta^* \in (-\infty, x_{nd}^*)$  such that  $\Sigma(x) < 0$  for  $x < \theta^*$ . In step 2, we have shown that it is not optimal to disclose an interval adjoint to the left of  $(\tau^*, +\infty)$ . Hence it cannot be true there exists a  $d < \tau^*$  such that  $\Sigma(x) < 0$

for  $x \in (d, +\infty)$ . It proves that an efficient disclosure law features  $D_m^* = (-\infty, \theta^*)$ , where  $\theta^* \in \mathbb{R}$ .  $\square$

**Proof of Proposition 5:**

If  $\phi(\cdot)$  is strictly concave, i.e.,  $\phi''(x) < 0$ , we have:

$$\begin{aligned} \frac{\partial \Sigma(x)}{\partial x} &= S\phi'(x_{nd}^*) - \phi'(x) > 0, \text{ for } x > x_{nd}^*, \text{ (from } S > 1 \text{ and } \phi''(\cdot) < 0), \\ \frac{\partial \Sigma(x)}{\partial x^2} &= -\phi''(x) > 0. \end{aligned}$$

We provide a counter-example. Assuming  $\phi'''(x) \geq 0$ , i.e.,  $\phi'(x)$  is strictly decreasing in a convex manner, which guarantees that  $S\phi'(x_{nd}^*) - \phi'(x) = 0$  always obtains a solution  $\hat{x} < x_{nd}$ , i.e.,  $\Sigma(x)$  is strictly convex function minimized at  $\hat{x}$ . Hence, there always exists  $\bar{\kappa} \in (-\infty, \tau^*)$  such that  $\Sigma(x) > 0$  for  $x \leq \bar{\kappa}$ , i.e.,  $\Theta^*(x) = 1$  for  $x < \bar{\kappa}$ .  $\square$

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