Modelling Eurozone’s Credit Default Swaps

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Abstract

Using Government balance sheet information we construct a structural CDS pricing model based on Merton’s contingent claims framework. We investigate whether in the absence of public releases regarding a Government’s financial position, the evolution of sovereign credit default swap spreads are driven by equity market volatility. We focus on spreads from European Union member states that have adopted the euro currency as these states have limited access to monetary policy tools to mitigate any deterioration in the Government’s finances.

Our results show that a contingent claims framework is not only able to model sovereign credit default swaps more accurately than a naive no-change model, its relative accuracy increases with the evaluation horizon.

Keywords: Sovereign risk; Credit default swap; Structural model

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1 Introduction

The recent events in Europe have heightened the importance of accurately quantifying sovereign credit risk for investors and regulators, both domestically and internationally. A credit default swap (CDS) is one type of instrument that enables easy transfer of credit risk. A CDS does not require any assumption about the benchmark risk-free rate and is thus seen as a clearer indicator of cross-sectional and time-series credit quality information than a bond (Longstaff, Mithal and Neis (2005) and Ericsson, Jacobs and Oviedo (2009)). However, despite a remarkable growth of single-name CDS contracts over the past decade, the sovereign CDS market is substantially smaller than the corporate CDS market, consisting of fewer obligors and representing only 18% of the total notional amount of CDS contracts outstanding as of June 2011.¹

Until recently, there was no perceived sovereign default risk in the developed economies, and consequently the liquidity in sovereign CDS spreads was concentrated in the emerging markets. Naturally, academic literature also predominantly assessed risks in emerging markets. However, the perception that advanced economies are immune to default has recently been challenged. Following the bailout of Lehman Brothers in September 2008 the CDS premiums on many Eurozone sovereign bonds began to widen. They widened again following the downgrade Greece’s (April 2010) and Ireland’s (April 2011) debt to junk status, increasing pressure on the solvency of these countries.

Motivated by the recent activity in the European sovereign CDS index and the sparse literature on sovereign credit risk, we apply a contingent claims analysis to the Government

¹ Data sourced from DTCC Trade Information Warehouse Reports, which can be accessed from: www.dtcc.com/products/derivserv/data/index.php
balance sheets of European Union (EU) member states to model Eurozone’s sovereign CDS spreads. We aim to add to the small body of literature on sovereign CDS, as to our best knowledge, this is the first paper to model CDS spreads of economic and monetary union economies.

Credit risk pricing models based on the contingent claims approach of Merton, known as structural models, have received considerable commercial attention in recent years. By providing insight into the fundamental determinants of default, structural models form the basis of a number of models used by practitioners to estimate probabilities of default. Although originally developed to estimate the probability of default of corporate firms, subsequent studies have extended the framework and applied the model to emerging market sovereigns (see, for example, Gray, Merton and Bodie (2007), Karmann and Maltritz (2002) and Oshiro and Saruwatari (2005)). However, the common European economic and monetary union arrangements prevent a direct transfer of existing emerging market models to EU sovereigns. In this paper, we extend the work of Allen, Rosenberg, Keller, Setser and Roubini (2002), Gray, Merton and Bodie (2007), and Gapen, Gray, Lim and Xiao (2005) by developing a generic Government balance sheet for EU member states which directly captures Government fundamentals and is thus suitable for estimating sovereign credit risk of countries bound by an economic and monetary union. Using the balance sheet estimates as inputs in a structural model, we investigate the degree to which CDS spread evolution can be explained by the evolution of equity volatility, in the absence of public releases of sovereign fundamentals.

The remainder of the paper is organised as follows. Section 2 reviews past and recent literature. Sections 3 and 4 describe the model and data respectively. Section 5 presents and discusses the results. Section 6 concludes.
2 Literature Review

2.1 Determinants of sovereign credit risk

Frank and Cline (1971) and Feder and Just (1977) exemplify a field of work that search for explanatory factors of developing country debt defaults using actual default data. These studies have identified a wide range of macroeconomic variables such as debt ratios, output ratios, growth and institutional measures that produce statistically significant explanatory power, yet only marginal predictability. Berg and Sachs (1988) claim that many of the macroeconomic variables are merely symptoms of a debt crisis, and therefore may have explained the poor predictability of earlier studies.

Rather than examine actual defaults, another line of research has focused on the changes of traded sovereign risk premiums. Early empirical studies on the pricing of bonds and syndicate loans\(^2\) include Edwards (1983, 1985) and Boehmer and Megginson (1990). Edwards (1983, 1985) report that of all the variables under examination, only public debt ratios and international reserves ratios are significantly related to higher risk premium, suggesting that international lenders take into account some country specific factors. Boehmer and Megginson (1990) demonstrate that a country's solvency measures have stronger relationship than its liquidity measures, suggesting that international lenders are less concerned about temporary cash shortages. In an intertemporal analysis, Eichengreen and Mody (2000) find that market sentiment rather than the fundamentals is a large driver of changes in spreads over short periods.

Following the standardisation of CDS contract by ISDA, the global market for single-name CDS has grown dramatically. Nevertheless, sovereign CDSs still represent a small proportion of the entire CDS market and are also small relative to the market for sovereign

\(^2\) Loans extended to less developed countries (LDCs)
bonds. (Duffie, 2010). Figure 1 provides a comparison of the size of sovereign CDS market for the selected Eurozone countries as of the end of June 2011.

[Insert Figure 2 here]

The basic features of a sovereign CDS are identical to a corporate CDS. In a typical transaction, the buyer of protection makes a series of payments to the seller of protection and, in return, receives payment if the sovereign entity over which the CDS is written experiences a credit event such as bankruptcy, failure to pay or restructuring. In the event of default by the reference entity, the contract is either settled in cash or by the delivery of a qualified bond. For a standard sovereign CDS, the three credit events that trigger the payment as defined by the International Swaps and Derivatives Association (ISDA) are, failure to pay, restructuring and repudiation/moratorium.

Having been shown to be a superior measure of issuer credit risk than bond yield spreads, several authors (see, for example, Ericsson et al. (2009a) and Longstaff, Pan, Pedersen and Singleton (2011)) re-examine whether sovereign credit risk is driven by country specific or global factors. Results show that, consistent with earlier studies, a high proportion of sovereign CDS spreads is explained by global factors than country specific economic fundamentals. However, due to data limitations, the study only examined emerging markets.

A common feature of these studies is that the debt servicing capacity of a country rather than the Government is being considered. Although studies such as Karmann and Maltritz (2002) and Clark (2003) use country risk and sovereign risk interchangeably, largely based on the interdependence between the emerging market's entire economy and its public sector finance, we argue that country level macroeconomic variables may be too aggregated and consequently mask the true weaknesses of the public sector. Furthermore, variables such as GDP growth and balance of payment can be reported with considerable lags.
Consequently, we define sovereign risk as the risk of a Government defaulting or restructuring its debt and focus specifically on Government fundamentals through a public sector balance sheet. Our definition is consistent with Duffie, Pedersen and Singleton (2003) and with the definition of a credit event on sovereign CDS by ISDA.

2.2 Sovereign credit risk and the Merton model

Merton (1974) pioneered the basic structural approach to assessing credit risk by directly applying the theory of European options pricing developed by Black and Scholes (1973). The model relates credit risk to the capital structure of a firm to produce a forecast of the firm’s probability of default at a given point in time. Due to the model’s simplicity and economic appeal, numerous studies have extended its application to measuring emerging market sovereign credit risks. For example, based on the set up of Clark (1991), who shows that the ‘market value’ of a country can be measured by the present value of a country’s net exports, Clark and Kassimatis (2004) find that risk premiums implied from macroeconomic variables have statistically significant impact on the performance of stock returns. Further, Karmann and Maltritz (2002) show an anticipation of sovereign defaults by their estimated risk indicator. Oshiro and Saruwatari (2005) present a novel approach where an equity index is used as a proxy for a sovereign’s “equity value”. However it may be argued that such a method lacks economic underpinning.

Besides quantification of sovereign credit risk, two studies apply the structural model framework to pricing emerging market sovereign bonds. Hui and Lo (2002) model the credit spreads of Korean and Brazilian Government bonds by defining a signalling process (such as exchange rate) as a trigger of default. Moreira and Rocha (2004) present a two-factor structural model to explain the credit spreads of Brazilian Government bonds.
More in line with our study, Gray et al. (2007), quantify sovereign credit risk using a structural framework with a sovereign balance sheet. In their approach, a Government’s foreign-currency debt is treated as equivalent to a corporation’s liability, while its domestic-currency debt is treated like equity. Their argument is that a Government has the flexibility to issue, repurchase or dilute local-currency in a way similar to corporate equity. Using the proprietary MfRisk model, the paper shows that the modelled risk indicators track emerging country CDS spreads reasonably well.

2.3 Willingness to pay and volatility

Catão and Sutton (2002) identify two problems associated with studies on sovereign credit risk, the first being the non-economic factors influencing default decisions. While corporations are bounded by national bankruptcy laws, the decision to default by a Government can be both economical and political. Duffie et al. (2003) maintain that without a formal bankruptcy framework, a Government will default when it is optimal to do so by weighing up the benefits and the costs, namely, the loss of reputation (Eaton and Gersovitz, 1981), trade blockages, and future inability to borrow (Grossman and Huyck, 1989). In other words, default is determined endogenously. The problem, also known as willingness-to-pay, is difficult to account for in econometric models.

Leland (1994) and Leland and Toft (1996) model corporate credit spreads by assuming a firm becomes insolvent when its assets hit an optimal default triggering barrier that maximises the value of the firm. Kulatilaka and Marcus (1987) have attempted to model strategic default decisions by sovereigns. Through their model, they find that the decision to default strengthens with the variance of the GDP process and weakens when there is penalty to default. Further, Clark (2003) propose a model where the decision to default is analogous to an American option. At each point in time, the authority makes a decision and ‘exercises’
the option when it becomes optimal. Although theoretically appealing, the calibration of these models remains difficult.

As we focus on developed economies, we do not consider the issue of willingness-to-pay. Historically, defaults due to unwillingness-to-pay had occurred during revolts and violent transition of Governments, for example, China in 1949 and Cuba in 1960. Karmann and Maltritz (2002) argue that, willingness-to-pay has become less relevant, as closer integration of economies increases the threat of international punishment, thereby reinforcing the duty to meet debt obligations.

The second issue recognised by Catão and Sutton (2002) is that second moment measures such as volatility can have strong explanatory power for the Government's ability to service debt, yet are overlooked in traditional macroeconomic literature. Using a dataset of 25 emerging countries, Catão and Sutton (2002) find a strong positive relationship between external volatility (terms of trade), and policy volatilities (fiscal, money supply and foreign exchange control) with sovereign default. More recently, Hilscher and Nosbusch (2010) also report that volatility in the terms of trade is both statistically and economically significant in explaining sovereign spread variations. These results add further support to using Merton model since volatility is a key driver of credit spread in the model.

3 Model Description and Parameter Setting

3.1 Structural Credit Risk Model

The Merton (1974) model assumes the total value of the firm’s assets, \( A \), follows a geometric Brownian process as follows.

\[
dA = \mu A dt + \sigma A dW
\]  

(1)
where $\mu$ is the drift, $\sigma$ is the volatility of assets, and $dW$ is a standard Weiner process. The underlying assumption is that debt, $B$, can be represented by a zero-coupon bond with maturity at time $T$, and default occurs when the asset value is below the bond’s principal value at maturity. In this specification, equity, $S$, essentially represents a European call option on the assets of the company with the same maturity as the bond and a strike price equal to the face value of the debt, $D$. The value of the firm’s debt is equal to the value of a default-free discount bond minus the value of a put option written on the firm, with a strike price equal to the face value of debt and a time-to-maturity of $T$. It follows that the value of defaultable debt, $B$, at time $t$ can be expressed as:

$$B_t = D e^{-rt} - [D e^{-rt}N(-d_2) - AN(-d_1)],$$

where the first term in the equation is the value of the default-free debt and the expression enclosed by square brackets is the theoretical put option value. $r$ is the instantaneous risk-free rate, $N(\cdot)$ is the cumulative standard normal distribution function, and $d_1$ and $d_2$ are given by

$$d_1 = \log \left( \frac{A}{D} \right) + \left( r + \frac{1}{2} \sigma^2 \right) T$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

Since the risk-free rate is easily observable, and by equation 2 we have further obtained the value of a defaultable debt, we can easily solve for the credit spread, $s$, between the two zero-coupon bonds. From the results above, the value of a defaultable debt can be expressed as

$$B_t = D e^{-(r+s)t}$$

By substituting equation 2 into equation 3 and rearranging equation 3, Hull, Nelken and White (2004) show that the credit spread implied from Merton’s model can be expressed as
\[ s = -\frac{1}{t} \log \left[ N(d_2) + \frac{A}{D e^{-rt}} N(-d_1) \right] \] (4)

The model has several important properties. Firstly, a deterioration of the asset to debt ratio will increase the probability of default non-linearly, as shown in Figure 2(a). Furthermore, an increase in the asset volatility brings the asset value closer to the default barrier and thus also raises the probability of default in a non-linear way as shown in Figure 2(b). By accounting for non-linearity, the Merton model improves existing linear regression methods, for example, Bayoumi, Goldstein and Woglom (1995) who concede that yield spreads of US municipal bond market are related to fiscal variables in a nonlinear way.

[Insert Figure 3(a) and 3(b) here]

3.2 Sovereign balance sheet and other inputs

In most applications, the theoretical CDS spreads in the Merton model are a function of easily observable variables (equity price, debt per share, risk-free rate), and asset value and its volatility, which although unobservable, can be inferred from the equity market via Ito’s Lemma. However, in contrast to the original model specification designed for listed firms, Governments do not have traded equity, nor are they obliged to produce financial statements, therefore requiring some subjective estimation of their assets and liabilities.

Our sample of EU member states that have adopted the common currency, the Euro, further complicates the issue. Unlike most developing countries previously examined, the central banking role in the euro area is performed by the Eurosystem, which is comprised of the European Central Bank (ECB) and national central banks (NCB) of the EU member states. The Eurosystem is in charge of monetary and exchange rate policies, in which the NCBs implement the policies under the directorship of the ECB. Reserve management is conducted by the NCBs with some degree of independence, but for large operations, approval
from the ECB is required. Fiscal policy, however, is performed independently by each member state.

The first problem in recreating a sovereign balance sheet is that whereas the base money for emerging countries represents a domestic liability for the public sector, in the European Economic and Monetary Union, both NCBs and the ECB are legal issuers of the Euro currency. The amount on issue is determined by the share of each country’s capital with the ECB. Due to the common currency, it becomes difficult to measure the actual amount in circulation in any particular member country, as a holding by a resident may represent claim on foreign member state Governments.

[Insert Table 1 here]

Secondly, column 1 of Table 1 shows a high proportion of Government debt that is denominated in Euros. Accordingly, a seniority structure akin to Gray et al. (2007) cannot be established. It is also hard to infer seniority structure from the ownership of debt, as the decision about whether sovereigns would default on external or domestic debt is largely a political question (see, for example, Vasishtha (2010) and Roubini and Setser (2004)). As such, we refrain from making any generalisations regarding the seniority of debt in our model.

In order to produce meaningful measures of assets and debt, we supplement the conceptual sovereign balance sheet proposed by Currie and Velandia (2002) with actual observable data from Eurostat. In Currie and Velandia (2002), sovereign assets consist of the present value of fiscal revenue, foreign reserves and marketable securities, while liabilities consist of the present value of fiscal expenditures, public debt and contingent liabilities. Assets such as buildings and land are excluded from the balance sheet, because they do not

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5 Usually, external debt refers to debt held by non-resident while domestic debt is held by resident. Other definitions include the legal definition of “governing law” see Roubini and Setser (2004).
have a direct influence on the Government's ability to meet debt service. Figure 4 presents the structure of our modified balance sheet.

Figure 4

<table>
<thead>
<tr>
<th>Assets:</th>
<th>Liabilities:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial assets</strong></td>
<td><strong>Gross general Government debt</strong></td>
</tr>
<tr>
<td>• foreign reserves</td>
<td>• currency and deposits</td>
</tr>
<tr>
<td>• securities other than shares</td>
<td>• securities other than shares</td>
</tr>
<tr>
<td>• loans</td>
<td>• loans</td>
</tr>
<tr>
<td>• shares and other equity</td>
<td></td>
</tr>
<tr>
<td><strong>PV of Government revenue</strong></td>
<td><strong>PV of Government expenditure</strong></td>
</tr>
<tr>
<td>• taxes</td>
<td>• intermediate consumption</td>
</tr>
<tr>
<td>• social contribution</td>
<td>• wages</td>
</tr>
<tr>
<td></td>
<td>• fixed capital formation</td>
</tr>
<tr>
<td></td>
<td>• interests</td>
</tr>
<tr>
<td></td>
<td>• social benefits</td>
</tr>
<tr>
<td></td>
<td>• Government transfers</td>
</tr>
</tbody>
</table>

On the assets side, financial assets include foreign reserves, securities other than shares (e.g. private sector bonds), loans, and shares (e.g. public ownership in corporations).\(^7\) Government revenue is comprised mainly of taxation revenues and social contribution (payment made by employers to the social security schemes). On the liabilities side, general Government debt is made up of currency and deposits (notes and coins issued by the NCB plus deposits) and securities other than shares and loans. Government expenditures comprise of intermediate consumption (goods and services consumed as input for production), wages,

\(^7\) Full definition of ESA95 accounting categories can be viewed from http://circa.europa.eu/irc/dsis/nfaccount/info/data/esa95/en/esa95en.htm
fixed capital formation (investment in fixed assets), debt interests, social benefits, and Government transfers (current and capital transfers).  

We exclude contingent liabilities from our specification, which arise for example when financial institutions deemed as too-big-to-fail require Government funding to recapitalise. Gray et al. (2007) put forward measuring contingent liabilities as a put option on the financial sector assets. However, the added benefit of producing such point estimates is questionable given the number of additional modelling assumptions.

Finding the present value of future fiscal revenue and expenditure is nontrivial. No empirical studies that we know of has carried out such quantification. As a result, we apply a simple perpetuity model. We use 10-year Government bond yield as the discount rate for calculating the present value of Government revenue and expenses owing to greater liquidity and further assume that the growth rate is negligible. Regardless of our choice, it can be shown that the choice of discount rate applied has limited impact on the spreads.

By the nature of perpetuity, revenues and expenditures constitute a large part of Government assets and liabilities respectively. This implies that the ability of sovereigns to service debt will depend largely on the balance between receipts and expenses. This is supported by Haugh et al. (2009) who find that during periods of greater risk aversion, interest payment on Government debt and the forecasted fiscal deficit are both important factors in explaining Government bond yields. In addition, Bernoth, Hagen and Schulknech (2006) maintain that the ratio of Government interest expense to revenues is "closer in spirit to measures of borrower quality commonly used in corporate finance", and that "it focuses on

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8 General Government includes central Government, public corporations and if applicable state, local Governments and social security funds. Public corporations are defined as whose sales are not able to cover at least 50% of their costs. (Eurostat, 2011)
9 See Brealey, Allen and Myers (2006 p.65)
10 It can be shown that if the present value of revenue (R) and expenditure (E) account for a large component of assets and liabilities, the leverage in Merton model may be proxied by log(R/r)-log(E/r), which is simplified into log(R/E)-log(r)+log(r). The discount rate effectively cancels out.
the constraint high debt burdens impose on the annual budgetary flows." Similarly, Beers and Cavanaugh (2006) contend that Government debt burdens do not necessarily correlate with credit ratings, rather it is the ability to support the debt levels that matters most. Consistent with these studies, our implementation assigns a heavier weight to Government revenue and expenditure.

In the original model specification, for corporations with traded equity, asset volatility is derived as a linear approximation of the equity volatility. However, as there is no equivalent measure in Government’s capital structure, we consider a number of alternatives. Money supply and foreign exchange volatility for each country considered by Gray et al. (2007) and readily utilised in emerging economies literature cannot be determined in our sample due to the common currency and a centralised monetary policy. Volatility of the economy, as measured by GDP, or any other measure that drives the changes in fiscal revenues are also problematic as they are only compiled annually and thus cannot produce timelier default indicators of credit risk.

Consequently, we estimate asset volatility using the index equity market volatility. Our reasoning is based on data availability and finding by Dieckmann and Plank (2011) who report significant positive association between stock market volatility and sovereign CDS spreads. Westphalen (2001) also reports positive and significant impact of local equity market volatility on the short term bonds. Intuitively, equity volatility can arguably be seen as measure of the volatility of received taxes.

Figure 5 plots a 10 year rolling annual stock index volatility against GDP volatilities of 24 OECD countries from 2000 to 2010. As expected, GDP is less volatile than the stock market, but there is a clear positive association between the two volatility measures.

[Insert Figure 5 here]
As the model inputs are released at different frequencies, we assume data released at lower frequencies remains constant between updates. Subsequently, we obtain model implied credit spreads by solving equation (4).

3.3 Model calibration

We introduce a parameter $\gamma$ to dictate the proportion of debt that enters into the model, and is analogous to an uncertain default barrier used by the CreditGrades CDS pricing model (for CreditGrades see Finger et al. (2002)).

By introducing a variable $\gamma$ into our model we are able to calibrate the proportion of debt used in the estimation of spreads to ensure that the model spreads match the observed CDS spreads at the beginning of each quarter following the release of Government finances. Between updates model spreads are priced using most the recent value of $\gamma$. The rationale for augmenting the amount of debt used to estimate spreads in the model is as follows; Firstly, debt can be refinanced before it becomes due and thus only a proportion of total debt should enter the calculation. Secondly, Jones, Mason and Rosenfeld (1984) tested the Merton model and found that the credit spreads on corporate bonds were too high to be matched by the diffusion process suggesting the model does not price the default risk sufficiently. Furthermore, due to the instantaneous default probability of a healthy firm being zero under a continuous process, the diffusion approach predicts that the term structure of credit spread should always start at zero and slope upward for firms that are not currently in financial distress. Using a variable $\gamma$ therefore corrects for the potential underestimation of spreads. Thirdly, upon the release of Government finances to the market, traders hold complete information about the Government’s finances and impound it into the spreads. As our focus is on the evolution of spreads in the absence of Government provided information we implicitly assume that when information is released to the public CDS spreads reflect the true probability of default.
4 Data

Our sample period extends from 3 July 2007 to 30 June 2011. The start of the sample coincides with the first decline in value of high grade mortgage backed securities in the United States, raising concern over the valuation of these products, which subsequently led to widespread market panic by August 2007 (Brunnermeier, 2009). Furthermore, prior to July 2007, the Western European sovereign CDS market exhibited little liquidity. Although the euro was utilised by 16 countries at the start of the sample, we exclude Vatican City, San Marino, Luxembourg, and Monaco from the sample due to their small economy and unavailability of CDS spreads. Our sample thus contains the data for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherland, Portugal, Slovenia and Spain. For each of those countries we collect the following data:

4.1 Sovereign balance sheet data

Our data for sovereign assets and liabilities is obtained from Government Finance Statistics (GFS) released by Eurostat. The advantage of Eurostat database is that it adopts a common accounting method known as the European System of National Accounts (ESA95), and therefore provides better comparability across countries. GFS is a comprehensive database which covers Government cash flows as well as stock measures. Information extracted from the database includes total revenue, total expenditure\(^{11}\), financial assets, and general Government debt, all produced quarterly. Revenues, expenditures, financial assets and Government debt are reported quarterly. According to the ESA95 manual, valuation of financial assets and liabilities are based on their market values.

\(^{11}\) Revenue and expenditure are not seasonally adjusted by Eurostat, therefore, we apply the X12-ARIMA Seasonal Adjustment Program from the US Census Bureau.
Columns two to seven of Table 1 compare cross-sectional country fundamentals from 2006 and 2010. The table shows that the level of gross Government debt to GDP had increased between 2006 and 2010 in all member states. Furthermore, by 2010, only Slovenia and Finland were within the 60% boundary set by the Maastricht Treaty. In terms of expenditure, Ireland recorded the largest increase, which is likely due to bailing out failing financial institutions. In contrast, there is no clear trend in Government revenue.

4.2 Market data

To construct volatility, for all countries except Slovenia, we obtain from Datastream daily stock index price data. Slovenia's main equity index, SBITOP, was downloaded from the Ljubljana Stock Exchange website. To eliminate short-term noise and avoid potential daily market anomalies, we calculate Wednesday-Wednesday weekly average returns consistent with Gerlach, Schulz and Wolff (2010). Asset volatility is estimated using 2-year and 5-year historical index returns. 10-year Government bond yield and 5-year sovereign credit default swaps are also obtained from DataStream. In line with previous research, we select contracts with a 5-year maturity because they are the most commonly traded contracts. (Ericsson, Jacobs and Oviedo, 2009b) For the same reason, the CDS spreads are also averaged on a weekly basis. Table 2 provides basic statistical descriptions for the three market variables.

The final inputs into our model (equation 4) is summarised below:

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5 Results

We perform two sets of evaluations on our model. Firstly, we analyse the performance of the model on country-by-country basis aggregating weekly one-quarter ahead spread forecasts across the 5-year horizon, producing 16 individual spread forecasts for each country. Secondly, we investigate the average performance of the model across all countries at each step following the quarterly model calibration, producing 12 different forecasts. To evaluate our model’s ability to fit the actual spreads we use a range of commonly used metrics, including mean absolute percentage error (MAPE), root mean square error (RMSE), mean absolute error (MAE) and the Theil U-Statistic (Theil, 1966). We also assess the accuracy of our model against a naive estimate, a simple no-change model.

Based on the tabulated statistics in Table 3, it is clear that there is a substantial variation in results across the twelve countries. Our sample period spans unprecedented deterioration in sovereign finances marked by dramatic increases in sovereign CDS spread levels ranging from 1.1 basis points for Germany in July 2007 to 2226 basis points for Greece in June 2011. For example, Finland and Germany’s spreads were quite low and reasonably stable during the sampling period, and therefore modelling spreads in these countries

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value/Proxy</th>
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<tbody>
<tr>
<td>T</td>
<td>5 year</td>
</tr>
<tr>
<td>A</td>
<td>Financial assets + PV revenue</td>
</tr>
<tr>
<td>D</td>
<td>(Debt + PV expenditure)*γ</td>
</tr>
<tr>
<td>r</td>
<td>10 year German bond yield</td>
</tr>
<tr>
<td>σ</td>
<td>2 and 5 year stock index volatility</td>
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</table>
produces small RMSE, MAE statistics. In contrast, Ireland, Italy, Portugal and Spain were characterised by frequent swings and large increases in spreads over the sample period and our model for these countries exhibits very large RMSE and MAE statistics. Greece, on the verge of bankruptcy by June 2011 displayed the largest variation in spreads and thus our model provides a particularly bad fit using these metrics. Interestingly, using the MAPE statistic Austria, Belgium, France, Germany and Italy performed poorly compared to Greece and Ireland, reflecting the differences in the difficulty in fitting our model across the different spread levels.

An evaluation across the two models clearly shows that our model is better at capturing the evolution in the sovereign CDS spreads in the absence of balance sheet information released by the Government for all countries examined using the RMSE and MAE statistic. Our model is also preferred for all countries besides Germany and Slovenia when evaluated using the MAPE. Using the Theil U-statistic we find that the number is quite low for all countries, suggesting the model is able to fit the level of CDS spreads reasonably well. Separating the three components of the U-statistic into the bias, variance and covariance measures, we observe that model exhibits low bias with most of the error being generated by the remaining unsystematic errors.

Figure 6 displays the aggregate n-step ahead evaluation across the twelve countries using the RMSE statistic. As expected, based on the results from the previous table, the naive model provides an inferior fit to the contingent claims model. More importantly though, although the accuracy of the model decaes with the distance from quarterly calibration, the performance of our model relative to the naive model actually improves as the horizon increases. This indicates that changes in the equity volatility correctly signal the direction of
the evolution of spreads. The percentage improvement in the RMSE of our model to the RMSE of the naive model ranges from 15% for the one-step ahead forecast to 35% for the 12-step ahead forecast.

6 Conclusion

We propose a structural model, which relates the capital structure of a Government to sovereign credit risk. By modifying the balance sheets of EU member states and using them as inputs in a contingent claims framework along with equity market volatility, we are able to estimate the evolution of sovereign CDS spreads between quarterly releases of Government balance sheet information.

From our results of twelve countries in the euro area it is clear that our structural model consistently outperforms a naive model in fitting sovereign CDS spread level across the sample period from July 2007 to June 2011.
References


Figure 1

Figure 1 contrasts the evolution of sovereign CDS spreads for Germany, Greece and Portugal during the crisis period from July 2007 to June 2011. Key events in the credit market are marked by vertical bars.
Figure 2 ranks the sovereign CDS reference entities by their gross notional amount as of the end of June 2011. The right axis shows the number of contracts in the market.

Source: The Depository Trust & Clearing Corporation - Trade Information Warehouse Reports
Figure 3(a) plots the non-linear relationship between leverage (asset to debt ratio) and credit spread by holding asset volatility at 20% and the risk-free rate at 5%. Figure 3(b) plots the non-linear relationship between asset volatility and credit spread by holding leverage at 60% and the risk-free rate at 5%.

Figure 3(a) plots the non-linear relationship between leverage (asset to debt ratio) and credit spread by holding asset volatility at 20% and the risk-free rate at 5%. Figure 3(b) plots the non-linear relationship between asset volatility and credit spread by holding leverage at 60% and the risk-free rate at 5%.
Figure 5

Figure 5 depicts the positive relationship between GDP volatility and equity market volatility by plotting 10-year rolling historical annual stock index volatility against GDP volatility of 24 OECD countries from 2000 to 2010.
Figure 6 displays the aggregate 1 to 12-step ahead evaluation across the twelve counties using the RMSE measure. The lower the error, the better the fit.
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Source: Eurostat – Government Statistics

Column 1 shows the proportion of gross Government debt denominated in Euros. Data for Austria, Finland, France and Greece are not available. Columns 2 to 7 compare respectively the changes in Government debt, expenditure and revenue from 2006 to 2010. The figures are expressed as a percentage of GDP.
Table 2 provides basic statistical description of weekly estimates of stock index volatility, 10-year Government bond yield, and sovereign CDS spreads for each country.

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Table 3 Model Evaluation

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Table 3 tabulates the aggregate evaluations of the Merton and naive model 1- to 12-step ahead forecasts over a four year horizon from July 2007 to June 2011 assuming 5-year equity volatility estimates. 2-year equity volatility produces similar estimates. The Theil statistic is further broken down into a bias term, variance term and a covariance term. Both models are recalibrated quarterly.