Firm Fundamentals and Variance Risk Premiums

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Abstract

We develop a tractable valuation model which shows that future asset returns are predictably related to two firm characteristics, book-to-market (*bm*) and return on equity (*roe*), because these measures carry information about priced risk. The model we derive predicts a negative relation between expected variance returns embedded in option prices (variance risk premiums) and both *bm* and *roe*. We confirm this prediction using a variety of empirical specifications. Our results show that accounting-based characteristics simultaneously inform investors about cash flows as well as the priced risk of those cash flows.

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Keywords: Stock returns, Variance returns, Accounting-based Valuation, Risk

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I. Introduction

Investors face at least two types of uncertainty when choosing a security: the uncertainty about the return as captured by the return variance, and the uncertainty about the return variance itself. This latter source of uncertainty introduces an additional source of risk from holding assets. This so-called variance risk premium arises because investors generally dislike uncertainty about the return variance and, in equilibrium, demand a premium for accepting this risk (Bakshi et al., 2003; Todorov, 2009). Variance risk is an integral component of many financial assets. As a result, how investors price variance risk has fundamental implications for asset allocation decisions, the pricing of hedge derivative securities, and the behavior of financial asset prices in general (Cochrane, 2011).

Despite its importance to financial markets, we know very little about the cross-sectional determinants of variance risk premiums. Prior work has established that variance risk premiums exhibit large cross-sectional variation (Carr and Wu, 2009; Di Pietro and Vainberg, 2006). However, this work has not yet formally identified the drivers of this cross-sectional variation. Rather, it has suggested that because the factors which explain the cross-sectional variation in stock returns do not explain the cross-sectional variation in variance risk premiums, there is either a large inefficiency in the pricing of variance risk or else that the majority of variance risk is generated by an independent risk factor that the market prices heavily (Carr and Wu, 2009). The goal of our paper is to provide a formal and rigorous link between variance risk premiums and firm fundamentals, so that we can take a step towards better understanding the determinants of priced risk in traded variance.

We derive a valuation model that is consistent with efficiently functioning and rational markets and use this model to show that two firm characteristics, book-to-market (bm) and return on equity (roe), carry information about priced risk. The model relies on two main assumptions which are strongly supported by empirical data: that book-to-market has a finite unconditional mean and that the growth rate in book follows an auto-regressive “variance-in-mean” process.[1] Because the model ties *bm* and *roe* to priced risk, it shows that these characteristics should be systematically related to the expected returns of traded assets and provides an important and empirically testable prediction: there should be a positive (negative) relation between equity risk (variance risk) premiums and both *bm* and *roe*. This model prediction is important for several reasons.

First, a number of empirical studies have found that book-to-market and profitability measures are related to future stock returns (e.g., Ball et al., 2015; Daniel and Titman, 1997; Fama and French, 1992, 1993; Lyle and Wang, 2015; Novy-Marx, 2013). Our study shows that future stock returns are predictably related to *bm* and *roe* (a profitability measure) because these measures carry information about priced risk, and thus provides a rational risk-based explanation for the association between future stock returns and measures of book-to-market and profitability. Second, the model shows that the priced risk in traded variance is not independent of the priced risk in traded equity, but instead, that *bm* and *roe* carry information about systematic risk that impacts the risk premiums embedded in traded equity *and* variance. Third, because the positive (negative) relation of *bm* and *roe* with future stock (variance) returns can be predicted based on plausible and empirically supported assumptions, it is difficult to interpret our supporting empirical analyses as investor mispricing.

Our empirical analyses play a critical role in our paper because even with our solved model, the prediction that bm and roe are related to variance risk premiums is perhaps surprising given the empirical findings of Carr and Wu (2009) that variance risk premiums are not explained by factors that explain stock returns. Moreover, because variance contracts represent the price of a claim on future realized stock return variance and not underlying cash flows, the connection between slow moving levels-based variables such as *bm* and *roe* and the returns on traded variance, absent a solved model, is not *ex ante* obvious.[2] The fact that we find empirical evidence consistent with our model's risk-based

[1] We discus each of these assumptions in more detail in Section II.

[2] Even though prior work has established empirically that there is a negative association between aggregate equity returns and aggregate variance risk premiums, the relations with *bm* and *roe* that we derive do not depend on any *a priori* assumed relation between the equity and variance risk premiums.

predictions, despite the lack of an obvious relation, lies at the heart of our contribution. Without our model's predictions, it may be natural to attribute empirical associations of *bm* and *roe* with future variance returns to investor mispricing.

We confirm the predictions of the model in both cross sectional and time-series analyses. In our cross sectional tests, we find that the predicted negative relation between variance risk premiums and *bm* and *roe* only emerges when both *bm* and *roe* are included in the regression specifications. This finding is consistent with the model's prediction that *bm* and *roe* work together to capture the information about priced risk. We find that our conclusions are not attributable to known risk factors, as our results are unaffected when we control for contemporaneous stock returns, the full sample slope coefficients from the Fama and French (2015) five factor model, and an additional factor based on changes in the CBOE VIX. This result is noteworthy because in addition to a book-to-market factor, the Fama and French (2015) five factor model includes factor portfolios that are based on profitability, which is related to our measure of *roe*. This distinction echoes the central message of Daniel and Titman (1997) that characteristics themselves, as opposed to unobservable covariances with empirical factors based on those characteristics, carry information about future returns. We conduct a series of robustness tests to ensure that our results are not sensitive to measurement issues related to our sample composition, the holding period of variance returns, or the estimation of variance risk premiums.

We find similar results in our time series analyses. These analyses allow us to determine if firm specific news is driving our cross-sectional results and to investigate whether *bm* and *roe* carry information about priced risk in aggregate. In addition, this approach allows us to take a step toward linking these characteristics as economic drivers of aggregate variance and variance risk, something that Engle and Rangel (2008) suggest is sorely missing from this body of research. We use two different time-series specifications. First, we run a time series regression of the median variance return on the median *bm* and the

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In fact, the relation is only revealed once equity and variance prices are derived in equilibrium and the model predicts that *bm* and *roe* work in tandem to explain cross sectional variation in variance risk premiums.

median *roe* where each variable is calculated from the cross-sectional data on a monthly basis. Second, we run a similar regression, but replace the median variance return with the return on the variance of the S&P 500 index from Bollerslev et al. (2009). We find a strong negative relation between variance returns and *bm* and *roe* for both approaches, consistent with the model and with the results from our cross-sectional analyses. In addition, we find that the predicted associations between variance returns and *bm* and *roe* depend on the inclusion of both variables, suggesting once again that these variables work in tandem to explain variation in variance risk premiums.

The model, when coupled with our empirical tests, contribute to the broad literature that examines the relation between firm characteristics and asset returns (e.g., Ball et al., 2015; Van Binsbergen and Koijen, 2010; Daniel and Titman, 1997; Kelly and Pruitt, 2013; Lettau and Van Nieuwerburgh, 2008; Novy-Marx, 2013; Sloan, 1996; Piotroski, 2000). A number of recent studies have conducted empirical tests that have linked attributes of the firm, and in particular the profitability of the firm, to future stock returns. Fama and French (2006) find that profitability, measured by current earnings, is positively associated with average returns. Novy-Marx (2013) finds that profitability, measured by gross profits-to-assets, has roughly the same power as book-to-market in predicting the cross-section of average returns, and that conditioning on profitability increases the returns to value investing strategies. Hou et al. (2014) develop an empirical q-factor model consisting of the market factor, a size factor, an investment factor, and a profitability factor based on *roe* that largely summarizes the cross section of average stock returns. An open question is whether these empirical findings are consistent with efficiently functioning markets and if they hold under plausible assumptions. By deriving a tractable model, which is consistent with traditional asset pricing theory and relies upon assumptions that are consistent with observed empirical data, our paper shows that associations between measures of book-to-market, profitability, and future returns can be entirely consistent with rationally functioning markets. Our empirical tests provide new and compelling evidence that the predictable relation between these characteristics and priced risk is pervasive and that risk-based predictions are borne out in the data.

This paper is also related to the smaller literature that has developed analytical models that link expected stock returns with firm characteristics. Several papers have linked characteristics, such as firm size and the book-to-market ratio, to expected returns (e.g., Berk et al. 1999; Carlson et al. 2004; Gomes et al. 2003; Zhang 2005). We add to this literature by developing an approach that links two firm characteristics, *bm* and *roe*, to systematic risk which impacts both equity and variance risk premiums. Aside from offering new insight into an important area within academic research, this result also has important practical implications because we show analytically and verify empirically that these accounting-based characteristics can be used to trade variance, and hence that accounting information and fundamental analysis is useful for forecasting the returns of financial assets other than stocks.[3]

We also contribute to the literature that examines the sources of variation in variance risk premiums. Several prior studies have investigated this question from the aggregate market perspective. Bakshi and Madan (2006) link the market variance risk premium with higher order moments of the return distribution and investor risk aversion. Todorov (2009) shows that both past realized price jumps and stochastic volatility are important determinants of variance risk premiums. A smaller number of papers have investigated variance risk premiums at the individual security level. Han and Zhou (2012) find that stocks whose returns tend to be low when systematic volatility increases have higher variance risk premiums. We add to both the market- and firm-level studies by providing a parsimonious model to identify firm-level characteristics that are associated with the time series and cross-sectional variation in variance risk premiums. We show that these characteristics explain variation at both the firm level and at the market level.

Lastly, because the model we derive shows that firm fundamentals impact both the

[3] A small number of prior studies have examined whether various measures of historical variance of different accounting-based data can be used to predict future stock return variance and to construct option-based trading strategies (Goodman et al., 2011; Sridharan, 2015). However these studies do not relate accounting-based characteristics to variance risk premiums nor do they test the predictions of a model to offer insight into the broader question of whether these characteristics are associated with priced risk.

equity and variance risk components simultaneously, our study is also related to the extensive literature that examines the drivers of asset volatility. Prior studies have generally predicted volatility using time series information, rather than contemporaneous economic variables (Engle and Rangel, 2008). In a recent paper, David and Veronesi (2013) derive a model that relates variation in aggregate stock and bond prices to the earnings-to-price ratio. We add to this line of work by showing that accounting-based valuation models can be used to predict the returns of financial assets whose prices are based on stock return variance.

The remainder of the paper is organized as follows. Section II presents the model. Section III discusses the estimation of the model and outlines our data. Section III also provides our empirical analyses. Section IV provides robustness tests and also discusses and tests additional implications of our findings. Section V concludes the paper. All derivations are in the appendix.

II. The Model

In this section, we derive models that express expected stock and variance returns as linear combinations of *bm* and *roe*. Our derivation is similar to Van Binsbergen and Koijen (2010); Kelly and Pruitt (2013); Lyle and Wang (2015), but differs on an important dimension. These prior papers are agnostic about risk and assume that expected log-returns follow an exogenous AR(1) process. In contrast, we endogenize expected rates of return by solving a partial equilibrium model. This approach allows us to tie firm characteristics to the priced risk embedded in stock returns and stock return variance.

A. Main Assumptions

Our model relies on two main assumptions. First, we make the assumption that the log book-to-market ratio (*bm*) has a long-run mean that is time independent (i.e., it is a covariance-stationary process).[4]

$$\lim_{j\to\infty} E_t[\log(\frac{B_{t+j}}{M_{t+j}})] = \overline{bm} < \infty \quad (1)$$

where B_{t+j} and M_{t+j} represent the book value and market value of equity, respectively, at time $t + j$. This assumption is an expression about an accounting system that it is expected to be "value-relevant" and is broadly consistent with a number of other studies. For example, Pástor and Veronesi (2003, 2006) assume that at some time in the future, market values and book values converge because of competitive market forces. In addition, the popular Ohlson (1995) model assumes that abnormal earnings erode over time, which implies that market values and book values will be unconditionally connected in expectation. Moreover, if a firm is expected to remain a going concern and accounting systems are expected to become closer to "mark-to-market" through time, then a relation similar to (1) would be expected.

Second, we assume that the growth rate in book, $\log(\frac{B_{t+1}}{B_t}) \equiv g_{t+1}$, follows an auto-regressive "variance-in-mean" process,

$$g_{t+1} = \bar{g} + \kappa g_t + \eta\sigma^2_{g,t} + \sigma_{g,t}\epsilon_{t+1}. \quad (2)$$

Here κ is the persistence of book growth and η is the variance-in-mean coefficient which we solve for endogenously based on no-arbitrage conditions. The innovation term, ϵ_{t+1}, is assumed to be normally distributed with mean zero and variance one. This process assumes that growth rates are persistent, which is a common feature of this literature (e.g., Campbell, 1991; Nissim and Penman, 2001; Penman, 1991). We include η to allow for the fact that book growth rates may depend on conditional variance. This approach is similar to the "(G)ARCH in mean" models that have been used extensively in the finance literature to model asset returns (e.g., Engle et al., 1987; Glosten et al., 1993). These

[4] Chattopadhyay et al. (2015) find strong statistical evidence in support of this assumption using data from 29 countries. For a more comprehensive discussion of this assumption, please see Chattopadhyay et al. (2015).

models relate the dynamics of the firm's profitability to the operations of the firm. An example of this is an oil production company, where the producer's profitability would exhibit dynamics that resemble oil price dynamics. More generally, if firm profitability is related to fluctuations in the prices of the goods that the firms sells, and if those prices evolve in a manner that is consistent with a variance in mean process, then firm profitability will also follow a variance in mean process. This assumption of a variance in mean process is supported by prior work and borne out in the data. Arif et al. (2015) find that accruals (a component of book growth) have a significant negative relation with stock return volatility. In addition, we find strong empirical evidence that book growth is indeed related to book growth variance in our sample.[5]

To allow for time variation in the conditional variance of book growth, we assume that $\sigma_{g,t}$ follows the discrete time version of the popular Heston (1993) volatility model. Specifically,

$$\sigma_{g,t+1} = \omega\sigma_{g,t} + \gamma z_{t+1} \tag{3}$$

where γ is a non-negative constant and represents the "volatility of volatility", z_{t+1} is normally distributed with mean zero and variance one, and the covariance between z_{t+1} and ϵ_{t+1} is assumed to be q (i.e., $E_t[\epsilon_{t+1}z_{t+1}] = q$). By allowing z_{t+1} to be correlated with ϵ_{t+1}, we implicitly assume that investors use realizations in book growth provided in financial reports to update their estimates of the conditional variance of book growth. While not formally modeled in the paper, this relation is similar to a standard Bayesian learning model where investors learn about the volatility of book growth by observing realizations through time. Thus, large innovations in book growth lead to large revisions in beliefs about the variance of book growth.[6]

[5]Under no arbitrage, we find that the predicted relation between expected growth in book and conditional book growth variance is negative ($\eta = -\frac{1}{2}(\frac{1}{1-\kappa}) < 0$). Moreover, when we regress future roe on lagged roe, bm and future stock return variance (this relation is derived in the appendix and given in equation (54)) using the following specification: $roe_{t+1} = A_0 + A_1 roe_t + A_2 bm_t + A_3\sigma_t^2 + \epsilon_{t+1}$. We find that $A_1 = 0.578$, $A_2 = -0.011$ and $A_3 = -0.612$ and all are highly significant.

[6]To see that conditional variance depends on the history of book growth realizations, note that because $z_{t+1} \sim N(0,1)$ we can write it as $z_{t+1} = q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}$ where ϵ_{t+1} and ξ_{t+1} are uncorrelated

Aside from the above assumptions, we also adopt a stochastic discount factor, Λ_t, (the marginal rate of consumption for a representative agent in the economy) that prices all assets in the economy (e.g., Armstrong et al., 2013; Bakshi et al., 2003; Bakshi and Kapadia, 2003; Johnson, 2004; Pástor and Veronesi, 2003).

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \exp(-r_f - \frac{\sigma_\Lambda^2}{2} - \sigma_\Lambda w_{t+1}), \tag{4}$$

where r_f is the continuously compounded risk-free rate, σ_Λ is the volatility of the discount factor, and $w_{t+1} \sim N(0,1)$ represents random shocks to the state of the economy. The covariance between w_{t+1} and ϵ_{t+1} is assumed to be ρ (i.e., $E_t[\epsilon_{t+1}w_{t+1}] = \rho$) which we assume is positive. This implies that market values must then satisfy the no-arbitrage condition $M_t = E_t[\frac{\Lambda_{t+1}}{\Lambda_t}M_{t+1}]$. Our assumption about the dynamics of the discount factor are identical to that used by Armstrong et al. (2013), as well as (in discrete time) Johnson (2004) and Pástor and Veronesi (2003), and it generates expected returns consistent with the traditional (consumption) CAPM.

While the assumptions presented above are necessarily parsimonious to obtain closed form solutions, we show in the next section that they are realistic enough to generate stock return behavior observed in empirical data.

B. Market Values

As we show in the appendix, the above assumptions imply that the fair market value of a non-dividend paying firm in the economy is given by:

$$m_t = b_t + \alpha_0 + \alpha_1 roe_t - \alpha_2\sigma_{g,t}, \tag{5}$$

IID normal distributions. So an update in investors estimate of conditional variance is then given by $\sigma^2_{g,t+1} = (\omega\sigma_{g,t} + \gamma(q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}))^2 = (\omega\sigma_{g,t} + \gamma(q\frac{(g_{t+1}-E_t[g_{t+1}])}{\sqrt{\sigma^2_{g,t}}} + \sqrt{1-q^2}\xi_{t+1}))^2$, where the $\frac{g_{t+1}-E_t[g_{t+1}]}{\sqrt{\sigma^2_{g,t}}}$ term represents the normalized information in the "growth (i.e. earnings) surprise" that investors use to update their expectations about conditional variance.

where $\alpha_0 = -\overline{bm} - \alpha_1(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$, $\alpha_1 = \frac{\kappa}{1-\kappa} > 0$, and $\alpha_2 = \frac{\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-q\gamma}$.[7] This equation captures the intuition that the market value is equal to book value of equity plus a linear combination of a constant, return on book equity, and the conditional volatility of book growth. Higher return on book equity increases market value (α_1 is positive for all $\kappa \in (0,1)$), whereas volatility in book growth, $\sigma_{g,t}$, decreases market value (α_2 is positive for all $q < \frac{(1-\kappa)(1-\omega)}{\gamma}$ which is guaranteed if $q < 0$). The correlation coefficients ρ and q which tie growth and volatility to the state of the economy show how risk in book growth and the volatility of book growth impact market value. Intuitively, firms with innovations in profitability and risk that move more with the economy will have lower market values, because, all else equal, they have higher exposure to systematic risk. Indeed, equation (5) says that firms with book growth that is more highly correlated (ρ) with the state of the economy have lower market values. The same is true for the correlation coefficient q. Firms with conditional volatility that varies more with the state of the economy have lower market values. While the equity pricing equation matches well with economic intuition, it does not tell us how stock returns behave or how fundamentals relate to expected stock and other asset returns. Our next section shows the dynamics of stock returns and how they relate to firm fundamentals.

C. Risk Premiums

In this section, we first derive stock return dynamics and then use these dynamics to determine priced risk in both equity and variance markets.

C.1. Stock Returns

In the appendix, we show that stock returns (changes in log-stock prices) exhibit the following dynamics:

[7] A similar solution exists for dividend paying firms if dividends over the interval t to $t+1$ are proportional to either book value or market value.

$$r_{t+1} = \mu_t - \frac{1}{2}\sigma_t^2 + (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1} - \alpha_2\gamma z_{t+1}, \tag{6}$$

$$\mu_t = \underbrace{r_f}_{\text{risk free rate}} + \underbrace{\rho\sigma_\Lambda[(1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma]}_{\text{Equity risk premium}}. \tag{7}$$

Here μ_t is the expected rate or return on equity and $\sigma_t^2 (= E_t[(r_{t+1} - E_t[r_{t+1}])^2])$ is the conditional variance of the stock return. The coefficients α_1 and α_2 are defined above. The $-\frac{1}{2}\sigma_t^2$ term is an "adjustment" term because r_{t+1} represents a log (not a simple) return. Equation (6) suggests that the assumptions used to solve for market values deliver stock return behavior that is broadly consistent with how stock returns are believed to behave. Both discount rates, μ_t, and variances, σ_t^2, are time varying, which is consistent with the large literature in finance and economics (Cochrane, 2011; Tsay, 2005). Expected rates of return embody the intuition that higher risk, $\sigma_{g,t}$, in book growth increases the rate of return demanded by investors for holding the equity. The innovation terms are composed of shocks in book growth, $(1+\alpha_1)\sigma_{g,t}\epsilon_{t+1}$, and shocks in the volatility of book growth, $-\alpha_2\gamma z_{t+1}$. As a result, the model delivers return behavior that is consistent with the return decomposition literature (e.g., Campbell, 1991; Vuolteenaho, 2002). It shows that stock returns are a function of expected returns (μ_t), "cash flow news" ($(1+\alpha_1)\sigma_{g,t}\epsilon_{t+1}$) and "discount rate news" ($-\alpha_2\gamma z_{t+1}$). In addition, equation (6) captures the economic intuition outlined in Ball et al. (1993), that book growth (approximately earnings deflated by book) carry information about both cash flows and discount rates and equity prices move in response to both of these pieces of information.

Moreover, equity risk premiums, $\mu_t - r_f$, are a function of the priced risk in both cash flow and discount rate news. Specifically, because $\mu_t - r_f = -cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1}) - cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), -\alpha_2\gamma z_{t+1})$ this implies that expected equity returns carry information about priced risk to cash flows, $-cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1})$, and the priced risk in the risk of those cash flows, $-cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), -\alpha_2\gamma z_{t+1})$. This non-trivial result has an important implication. In a rational market, characteristics that carry information about priced risk in equities should also carry information about the priced risk in discount rates (or in the above model, stock return volatility itself).

C.2. Equity Risk Premiums as a Function of Fundamentals

Equation (7) shows that equity risk premiums are a function of the variance of book growth, an unobservable variable that must be estimated. In this section, we show that this latent variable can be substituted out using the market value equation which allows equity risk premiums to be expressed as a linear combination of firm fundamentals. We define the firm specific equity risk premium from the period t to $t+1$ as the continuously compounded return on market equity minus the risk free rate: $ERP_{t,t+1} = \mu_t - r_f$. We show in the appendix that, under no-arbitrage conditions, the expected equity risk premium can be written as:

$$ERP_{t,t+1} = \theta_0 + \theta_1 bm_t + \theta_2 roe_t, \tag{8}$$

where $\theta_0 = \theta_1(\alpha_0 - \alpha_2 q\gamma)$, $\theta_1 = (1-\omega) - \frac{\gamma q}{(1-\kappa)}$, $\theta_2 = \alpha_1\theta_1$. All of the constant terms are predicted to be positive if the correlation coefficient between book growth volatility and the state of the economy, q, is negative. This result is important because it extends the findings of Lyle et al. (2013) and formally shows that equity risk premiums are rationally associated with firm characteristics, and in particular they are increasing in both bm_t and roe_t. This suggests that prior studies which have documented a strong relation between future stock returns and these and other correlated variables are consistent with traditional asset pricing theory (e.g., Ball et al., 2015; Fama and French, 1992; Harvey et al., 2014; Novy-Marx, 2013; Kelly and Pruitt, 2013; Subrahmanyam, 2010).

While we do not formally model the accounting system that generates book values and earnings, the rationale for bm_t and roe_t carrying information about priced risk can be explained in the following way. Market values consist of realized cash flows (realized earnings) not paid out to stake holders, plus assets in place, plus expected discounted future cash flows. The sum of realized cash flows and assets in place represents book values, b_t. Information about future cash flows is captured by roe. Thus, once market values are combined with book values and return on equity, the remainder represents discount rates of future cash flows (expected returns).

C.3. Variance Risk Premiums as a Function of Fundamentals

The above result offers a rational explanation for the findings of prior empirical studies which link bm and/or profitability measures to future stock returns, but it does not tell us whether firm characteristics carry information about priced variance risk. Therefore, we next show that firm fundamentals are related to the risk embedded in stock return variance and that an expression for the firm's variance risk premium is also a linear combination of firm fundamentals. We define the expected variance risk premium as $VRP_{t,t+1} = E_t[\sigma^2_{t+1}] - R_f v_{t,t+1}$, where $v_{t,t+1}$ is the fair price for holding variance from t to $t+1$ and $E_t[\sigma^2_{t+1}]$ is the expected return variance over the interval t to $t+1$. In the appendix, we show that this expression can be combined with the stock return dynamics and the equation (5) such that the expected variance risk premium can also be written as a linear combination of bm and roe:

$$VRP_{t,t+1} = \phi_0 + \phi_1 bm_t + \phi_2 roe_t, \tag{9}$$

where $\phi_0 = \alpha_0\phi_1 + \eta_0$, $\phi_1 = q(1+\alpha_1)^2\omega\gamma((1-\kappa)(1-\omega) - \gamma q)$, $\phi_2 = \alpha_1\phi_1$ and $\eta_0 = -[(1+\alpha_1)\gamma q(1+\rho^2(\sigma^2_\Lambda + 1)) - 2\omega^2\alpha_2\rho\sigma_\Lambda]q\gamma(1+\alpha_1)$. Here, the key coefficients, (ϕ_1, ϕ_2), are predicted to be negative if the correlation coefficient, q, is also negative. Equation (9) offers an important and empirically testable prediction: if the two characteristics bm_t and roe_t have information about priced risk, then their relation with variance risk should be negative as long as the correlation between the volatility of book growth and the state of the economy (q) is negative. Given the large empirical evidence mentioned above which documents that the relation between bm_t, roe_t and future stock returns is positive, evidence of a negative relation between these characteristics and variance risk premiums would offer new empirical evidence that these characteristics carry information about priced risk.

III. Data and Empirical Analyses

This section describes the data collection process and the empirical implementation of the model presented in Section II. Our empirical analyses proceed in three steps. First, we examine whether the variance risk premium has a cross-sectional relation with bm and roe. We then examine whether there is an association between aggregate measures of variance risk and bm_t and roe_t in time series data to investigate whether bm_t and roe_t carry information about systematic risk.

A. Data

We collect stock price information from The Center for Research in Security Prices (CRSP), financial statement data from Compustat quarterly files, and option data from OptionMetrics. Our sample includes all firms with fiscal year ends of March, June, September, and December from January 1996 to December 2013. We require firms to have positive book values, at least four quarters of historical accounting information and beginning-of-month stock prices greater than $5. In addition, we require that each firm in the sample have sufficient options data to allow us to calculate variance contracts for both 30- and 60-day ahead expirations. Our final sample consists of 307,510 firm-month observations.

At the end of each month, we match a firm's most recently reported quarterly book value of equity and return on book equity to the price of variance contracts with standardized expiration of 30 and 60 calendar days ahead.[8] We estimate the cost of purchasing a variance contract using the model free method outlined in the appendix. Realized vari-

[8]To ensure that the financial statement data is publicly available at the end of the month, we use the firm's report date in Compustat (the RDQ variable) and add an additional month of time before the firm obtains a new book or earnings value.

ance is the sum of squared daily log returns. We then calculate variance risk premiums following Carr and Wu (2009) as the difference between future realized variance and the cost of purchasing a variance contract. All estimated and independent variables are winsorized at the 1% level. We use variance contracts on the S&P 500 Index in some of our tests. For these contracts, we use both the price of the variance contract and the realized variances on the S&P 500 Index employed by Bollerslev et al. (2009).[9]

Table's I and II provide descriptive statistics of key variables used in the analysis as well as other firm-level variables commonly used in cross-sectional asset pricing studies. Table I shows that the price of a 30 day ahead variance contract $v_{t,t+1}$ is on average greater than future realized 30 day variance as well as lagged variance, consistent with variance carrying a negative risk premium. Moreover, the economic magnitude of this premium is large, with the excess return on a 30 day variance contract averaging -16.72 percent. Realized stock returns in our sample average 0.77 percent per month, the log book-to-market ratio is -0.89 and quarterly rate of return on book equity is 1.38 percent. Log market cap ($size$) is 7.28 and β is 1.31, consistent with firms in our sample being large and having a high positive covariance with the overall market.

Consistent with intuition, Table II shows that the univariate correlation between the variance contract and realized variance (both future and lagged) is large and exceeds 0.5. The correlation between stock returns and the price and returns of variance is negative, but positively associated with both bm and roe. The return on variance is negatively related to bm and roe, and positively related to β, $size$ and lagged variance ($lvar$). Consistent with prior research, larger firms have lower stock return variance (both future and lagged).

[9]We thank Hau Zhou for making this data publicly available. The data can be found at: https://sites.google.com/site/haozhouspersonalhomepage/ .

B. Empirical Tests

B.1. Cross-sectional Tests

Our cross-sectional analyses follows directly from equation (9). We first write equation (9) in terms of a traditional risk premium as follows:

$$E_t[R^v_{t,t+1} - R_f] = \frac{E_t[\sigma^2_{t+1}]}{v_{t,t+1}} - R_f = \frac{\phi_0}{v_{t,t+1}} + \phi_1 \frac{bm_t}{v_{t,t+1}} + \phi_2 \frac{roe_t}{v_{t,t+1}}. \tag{10}$$

This leads directly to the following empirical specification:

$$R^v_{t,t+1} - R_f = a_0 + a_1 \frac{1}{v_{t,t+1}} + a_2 \frac{bm_t}{v_{t,t+1}} + a_3 \frac{roe_t}{v_{t,t+1}} + \xi_{t+1}. \tag{11}$$

The book-to-market ratio is calculated as $bm_t = \log(\frac{B_t}{M_t})$, where B_t is book value of equity from Compustat and M_t represents market capitalization calculated as stock price multiplied by shares outstanding from CRSP divided by 1,000. The return on equity is calculated as $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$, where x_t is income before extraordinary items from Compustat. $v_{t,t+1}$, as stated above, is calculated using the model free approach outlined in the appendix using a cross-section of firm level options from the OptionMetrics volatility surface file. ξ_{t+1} is a mean zero error term. (9) predicts that a_2 and a_3 will be negative.

For our first set of analyses, we estimate equation (11) monthly using the Fama-MacBeth approach. Table III provides the results of regressing variance risk premiums on each right had side variable separately, and then all simultaneously as specified in equation (11). Moving from left to right across the table it becomes clear that *bm* and *roe* work together to deliver the predicted relation for *roe*. While the coefficient on *bm* is negative and statistically significant when it is the only independent variable in the regression, the coefficient on *roe* is positive and significant when it is the only independent variable in the regression. When both variables are combined as prescribed by equation (11), the predicted negative relation emerges in the data. Both coefficients are highly significant. In addition, the explanatory power of the regression goes up considerably when the full set of variables are included. We test whether our results are sensitive to the holding period by repeating our analysis with 60 calendar day ahead variance returns. The results in Column (6) of Table III show that the coefficients of interest remain negative and significant. In addition, these coefficients and are roughly double in magnitude, which is the expected result from doubling the holding period.

We next examine whether the conclusions in Table III are sensitive to the inclusion of a set of additional control variables. We include three lagged return variables, $R_{t-1,t}$, $R_{t-12,t-1}$, $R_{t-36,t-13}$, which equal lagged monthly returns, lagged yearly returns excluding the most recent month, and lagged three year returns excluding the most recent annual return, respectively, to control for the known negative relation between stock returns and variance risk premiums. We include a proxy for jump risk, calculated as the difference between call and put option implied volatility (Yan, 2011), to control for the association between jump risk and variance risk premiums. Finally, we include controls for lagged monthly variance, estimated from daily stock return data, and size, which is equal to the log of market capitalization. The results in Table IV indicate that our conclusions are unchanged by the inclusion of these additional variables. Each of the new control variables is statistically significant and improves the explanatory power of the regression specification, but the coefficients on both *bm* and *roe* are very similar to those in Table III. Comparing Column (3) of Table IV with Column (5) of Table III, we can see that the coefficient on *bm* (*roe*) is -0.062 (-0.468) compared with -0.055 (-0.609). A similar pattern exists for the 60 day ahead specifications in Column (6) of Table IV and III. These results indicate that *bm* and *roe* are capturing a relation with variance risk that is distinct from the relation with lagged returns, lagged variance, size and jump risk.

B.2. Cross-Sectional Results Using S&P 500 firms

We repeat the above cross-sectional analysis after limiting the sample to firms which are constituents of the S&P 500 Index for two reasons. First, S&P 500 firms are large and have highly liquid stocks. Therefore, if our full sample results are driven by liquidity constraints or a market imperfection that affects small firms or thinly traded stocks, then the results would be different for the S&P 500 firms relative to our full sample. Second, options on S&P 500 firms are actively traded and thus the price of variance extracted from these options will contain less noise than the prices of variance extracted from options on firms with lower option trading activity.

Table V provides the results of the cross-sectional analysis using only S&P 500 firms. Despite the significantly reduced sample size, the empirical results conform with the findings from the full sample of firms. The coefficients on *bm* and *roe* are somewhat smaller in magnitude, particular for *bm*, but the strong negative association with variance returns remains. The fact that the predicted relation between *bm*, *roe* and variance returns is preserved for the biggest and most liquid firms in the economy is noteworthy because many empirical relationships between firm characteristics and stock returns either vanish or are significantly attenuated when samples are constrained to large firms (Fama and French, 2008).

C. Time Series Tests

We investigate whether the relation between variance returns and *bm* and *roe* is preserved in the aggregate using time series data to identify whether our results are attributable to latent systematic risk. The use of time series data to examine the drivers of aggregate volatility is common in the literature (e.g., Engle and Rangel (2008)). We conduct this analysis in two ways. First, we calculate the median variance return, median *bm* and median *roe* in the cross-section over our 1996-2013 sample period. We then run a time series regression of the median variance return on the median *bm* and the median *roe*. Second, we run a time series regression where we use the return on the variance of the S&P 500 index from Bollerslev et al. (2009) as an aggregate measure of the variance return. This extends our sample period back to 1990, as we are no longer constrained to only those firms with actively traded options. We then regress the return on the variance of the S&P 500 index on median *bm* and median *roe*.

Obtaining similar results in both cross-sectional and aggregate tests is not obvious. For example, Kothari et al. (2005) find that earnings surprises have a positive cross-sectional association with stock returns, but a negative association in aggregate tests. Similarly, Hirshleifer et al. (2009) find a strong positive relation between aggregate accruals and aggregate stock returns, which is the exact opposite to the findings of the cross-sectional results documented by Sloan (1996).

Table VI provides the time series regression results. As with our analysis in Table III, we show results separately and together for *bm* and *roe*. As with our main analysis, we find that the predicted associations between variance returns and *bm* and *roe* depend on the inclusion of both variables. In column (1), the coefficient on *bm* is positive and statistically significant. In contrast, the coefficient on *bm* is negative and statistically significant in both Column (4) and (5) when *roe* is included in the specification. This suggests that *bm* and *roe* work together to provide the predicted relation. The full specification in column (5) has negative and statistically significant coefficients for each variable, consistent with our model.

The analysis in columns (6) through (10) use the return on the variance of the S&P 500 index from Bollerslev et al. (2009) as the dependent variable. The results closely mirror those in columns (1) through (5). In column (6), the coefficient on *bm* is positive and insignificant. In contrast, the coefficient on *bm* is negative and statistically significant in both Column (9) and (10) when *roe* is included in the specification. Once again, this implies that *bm* and *roe* work together to provide the predicted relation. The full specification in column (10) has negative and statistically significant coefficients for each variable, consistent with predictions of the model.

Figures I and II illustrate the time-series data for the variance risk premium, *bm* and *roe*.[10] Figure I uses the median variance return, and Figure II uses the variance of the S&P 500 Index. Because the coefficient on the univariate regression of the variance risk premium on *roe* is negative, both figures show the negative of *roe*. These figures

[10]We multiply median *roe* by -1 for ease of exposition since it ensures that all the variables in the figure are on average negative.

illustrate visually how *bm* and *roe* work together to explain the variation in the variance risk premium. In particular, *roe* is quite persistent and so it captures the long term trend in the variance risk premium. Between 1998 and 2002 when variance risk premiums are, on average, higher than the surrounding years, the same is true for *roe*. Similarly, when variance risk premiums increase in 2007, so does *roe*. In contrast, *bm* seems to capture the short-term spikes in variance risk premiums quite well. The largest spike in *bm* occurs in 2008, around the time that variance risk premiums also have its largest spike.

D. Forward Factor Controls

Our next specification augments equation (11) to include the full sample forward looking slope coefficients from the Fama and French (2015) five factor model as well as the slope coefficient that captures co-movement with the CBOE VIX (Carr and Wu, 2009). We include these additional factor loadings, along with contemporaneous (R_{t+1}) stock returns, to ensure that the coefficients on *bm* and *roe* are not driven by an association with one of these known factors or by the negative association between stock returns and variance returns.

The results in Table VII indicate that our conclusions are unchanged by the inclusion of these additional variables. The first column regresses variance returns on the full sample slope coefficients using the Fama and French (1993) three factor model. The second column adds the two more recent factors related to profitability and investment, and the third adds the factor that captures co-movement with the CBOE VIX. The fourth column adds all of the control variables from Table IV. Across each of the first four columns, the coefficients on *bm* and *roe* are negative and highly significant, and are virtually unchanged when compared with Table III. This indicates that while significant, the additional explanatory variables do not affect the cross-sectional relation predicted by our model. We provide additional support for this conclusion by repeating our analysis using 60 calendar day ahead variance returns, the results of which are provided in columns (5) through (8).

E. Portfolio Sorts

The prior tests examine whether the relation between variance returns and *bm* and *roe* are statistically significant and not explained by associations with other firm attributes or risk factors. However, they do not offer insight into the economic magnitude of this relation. Our next set of analyses investigate the economic magnitude of the relation between variance returns and *bm* and *roe* by determining whether economically meaningful returns to a variance trading strategy based on *bm* and *roe* are present in the data. The results in Table VIII show that the variance return two-way portfolio sorts based on *bm* and *roe* map well into the predicted relation. The returns to this strategy are, on average, decreasing in both *bm* and *roe*. The variance return is -16.18 percent per month when both *bm* and *roe* are in the lowest quintile, compared with -24.06 percent when both *bm* and *roe* are in the highest quintile. The returns on a hedged portfolio within each *bm* quintile are all negative after the first quintile, and the magnitudes increase for higher *bm* quintiles.

The hedge returns generated from the returns net of the Fama-French three factors, Fama-French five factors, and Fama-French five factors plus the VIX factor provided in columns (7) through (9), respectively, are very similar to those using raw returns in column (6). The returns on a hedged portfolio within each *bm* quintile are all negative and the magnitudes increase for higher *bm* quintiles. The large and statistically significant α's for each of these specifications suggests that the inclusion of classic risk factors have virtually no impact on the average variance returns based on our strategy. This finding is consistent with Carr and Wu (2009), who also document that traditional risk characteristics do not explain variance risk premiums. We find a similar return pattern in Panel B, which uses 60 day ahead variance contracts. As with Panel A, the α's for each specification are negative and significant, and increase for higher *bm* quintiles.

IV. Additional Robustness Tests and Implications

A. Alternative Proxies for Variance Risk Premiums

Our main specification uses the difference between the sum of squared daily log returns and a variance contract as a proxy for variance risk premiums. This approach is consistent with prior work that focuses on realized variance (e.g., Carr and Wu (2009)). However, our model is based on expected variance risk premiums. Therefore, to ensure that our results are not sensitive to using realized variance as a proxy for expected variance risk premiums, we estimate conditional daily variances using a quadratic GARCH(1,1) (QGARCH(1,1)) model for each firm in our sample. This approach is a close approximation of the variance dynamics implied by our model. Our sample size is slightly smaller in these tests because the algorithm used to estimate the QGARCH model did not converge for all firms in our sample. We calculate variance risk premiums at the end of each month in two ways.

First, we accumulate future daily realized conditional variances estimated using the QGARCH model. Table IX presents the results of regressing 30- and 60- day-ahead variance risk premiums under this approach on *bm* and *roe* and the control variables from both Table IV and Table V. Columns (1) through (4) provide the 30 day-ahead results; columns (5) through (8) provide 60 day-ahead results. Consistent with our prior findings, the coefficients on *bm* and *roe* remain negative and highly significant and the associations are virtually unchanged when control variables are added.

Second, we forecast cumulative expected variance based on current conditional variance estimates and the QGARCH model's estimated parameters. Table X presents the results of regressing 30- and 60- day-ahead variance risk premiums under this approach. The specification mirrors that in Table IX. As before, columns (1) through (4) provide the 30 day-ahead results, and columns (5) through (8) provide 60 day-ahead results. Consistent with our prior findings, the coefficients on *bm* and *roe* remain negative and highly significant and are robust to control variables. Overall, the results in Table IX and X provide assurance that our conclusions are not sensitive to the approach we use to estimate expected variance risk premiums.

B. Implications for Book Growth

We provide additional assurance that our model assumptions are realistic by examining whether our second assumption, that growth rate in book follows a variance in mean process, is borne out in the data. To do so, we require an estimate of the expected variance of growth in book. We cannot estimate the variance of book growth firm by firm because of time series limitations (accounting numbers are only reported quarterly). However, the appendix shows that the no-arbitrage conditions imply that we can use accounting- and market-based variables to test this assumption. Specifically, the appendix shows that future return on equity can be expressed as a linear function of lagged roe_t, bm_t and expected stock return variance, σ_t^2 as follows:

$$roe_{t+1} = \varphi_0 + \varphi_1 roe_t + \varphi_2 bm_t + \varphi_3 \sigma_t^2 + \sigma_{g,t}\epsilon_{t+1}, \tag{12}$$

where $\varphi_0 = \bar{g} + \omega\gamma\alpha_0 + \frac{1}{2}\frac{1}{1+\alpha_1}\alpha_2^2\gamma^2$, $\varphi_1 = (1 - \frac{\omega\gamma}{1-\kappa})\kappa$, $\varphi_2 = -\omega\gamma$, and $\varphi_3 = -\frac{1}{2}(1+\kappa)$. The most important coefficients are φ_1, φ_2, and φ_3. The no-arbitrage condition implies that the coefficients on bm_t and σ_t^2 are negative and the coefficient on lagged growth will be positive so long as φ_2 is small in magnitude. Thus, if the model is reasonably descriptive for the average firm in our sample, then cross-sectional regressions of future roe on these variables should yield coefficients that are consistent with equation (12).

Table XI provides the cross-sectional estimates from monthly regressions based on equation (12). The results are consistent with our expectations. The coefficient on roe_t is positive and significant, and the coefficient on bm_t is negative, significant and smaller in magnitude than the coefficient on roe_t. We estimate the variance term using both realized variance and conditional variance estimates from a quadratic GARCH(1,1) model, as described in the prior subsection. In all cases, the coefficient on the expected stock return variance term is negative and highly significant, consistent with our expectation.

Overall, the results in Table XI confirm that our assumption that growth in book follows a variance in mean process is consistent with the data.

C. Additional Robustness Tests

We conduct several additional tests to confirm that our results are not attributable to persistence in the primary variables of interest. First, we mitigate the concern that our results are driven by auto-correlation in the cross-section of variance risk premiums by including up to six months of lagged variance risk premiums. In untabulated results, we find that relation between future variance risk premiums and *bm* and *roe* remain. Second, we mitigate the concern that our results are attributable to a seasonal component due to the use of quarterly accounting data by including up to four lags of *bm* and *roe* and also by conducting our analysis using annual instead of quarterly data. In untabulated results, we find that the negative relation between variance risk premiums and *bm* and *roe* remain.

V. Conclusion

This paper provides a tractable valuation model which shows that future asset returns are predictably related to two firm characteristics, book-to-market (*bm*) and return on equity (*roe*), because these characteristics carry information about priced risk. Because the model links *bm* and *roe* to priced risk, it shows that there should be a positive (negative) relation between equity risk (variance risk) premiums and both *bm* and *roe*. A large body of prior empirical work has found that book-to-market and profitability, which is related to *roe*, have a positive association with future stock returns. We find that *bm* and *roe* have a robust negative relation with future variance returns. These empirical findings are consistent with the model's predictions. Our results provide new and compelling evidence that the relation between these characteristics and priced risk is pervasive, consistent with the risk-based predictions of our model.

Overall, our analysis complements several prior studies by identifying firm-level characteristics that are associated with the time-series and cross-sectional variation in variance risk premiums documented in those studies (e.g., Todorov, 2009; Han and Zhou, 2012). It also represents an important contribution to the literature that examines the relation between firm characteristics and asset returns (e.g., Daniel and Titman, 1997; Haugen and Baker, 1996; Lewellen, 2014 among others). Our findings have important practical implications because we show that accounting-based characteristics can be used to trade variance, and hence that accounting information and fundamental analysis is useful for forecasting the returns of financial assets other than stocks.

The relation between *bm*, *roe*, and priced risk that we derive is based on, what we believe to be, plausible assumptions that are consistent with empirical data. However, we take accounting numbers as given and do not consider how the accounting system generates these numbers, *per se.* An interesting and potentially important future extension of this work would be to formally model an accounting measurement system to determine if ours or similar assumptions are consistent with accounting standards such as those prescribed by U.S. GAAP or IFRS.

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A. Derivations

A. Book-to-Market Derivation

Given the stationary assumption of bm, we have the following:

$$bm_t = \overline{bm} + \sum_{i=1}^{\infty} E_t[r_{t+i} - g_{t+i}], \tag{13}$$

where $r_{t+i} = m_{t+i} - m_{t+i-1}$ is the ex dividend return on market equity and book growth. As outlined in our text, the book growth process is given by,

$$g_{t+1} = \bar{g} + \kappa g_t + \eta\sigma_{g,t}^2 + \sigma_{g,t}\epsilon_{t+1}, \tag{14}$$

$$\sigma_{g,t+1} = \omega\sigma_{g,t} + \gamma z_{t+1}, \tag{15}$$

where $z_{t+1} = q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}$. $q \in [-1, 1]$ is a correlation coefficient and both ϵ_{t+1} and ξ_{t+1} are IID standard normal distributions. The ξ_{t+1} term is assumed to be uncorrelated with ϵ_{t+1} and the shocks to the discount factor w_{t+1}. To solve for bm_t we use the same approach as Bansal and Yaron (2004) and conjecture that the ratio is linear in the state variables g_t and $\sigma_{g,t}$, and verify that the solution satisfies the no-arbitrage condition

$$1 = E_t[\frac{\Lambda_{t+1}}{\Lambda_t}e^{m_{t+1}-m_t}], \tag{16}$$

$$= E_t[e^{\lambda_{t+1}+r_{t+1}}], \tag{17}$$

where $\lambda_{t+1} = \log(\frac{\Lambda_{t+1}}{\Lambda_t})$. We conjecture that the log book-to-market ratio is $bm_t = A_0 + A_1 g_t + A_2\sigma_{g,t}$, which implies that $g_{t+1} = r_{t+1} + A_1(g_{t+1} - g_t) + A_2(\sigma_{g,t+1} - \sigma_{g,t})$. Thus $r_{t+1} = g_{t+1}(1 - A_1) + A_1 g_t - A_2(\sigma_{g,t+1} - \sigma_{g,t})$. Since both λ_{t+1} and r_{t+1} are conditionally normal, then this implies that

$$E_t[r_{t+1}] + \frac{1}{2}V_t[r_{t+1}] = r_f - cov_t(\lambda_{t+1}, r_{t+1}), \tag{18}$$

where

$$E_t[r_{t+1}] = (1 - A_1)(\bar{g} + \kappa g_t + \eta\sigma_{g,t}^2) + A_1 g_t - A_2(\omega - 1)\sigma_{g,t}, \tag{19}$$

$$V_t[r_{t+1}] = ((1 - A_1)\sigma_{g,t} - A_2 q\gamma)^2 + (1 - q^2)A_2^2\gamma^2, \tag{20}$$

$$cov_t(\lambda_{t+1}, r_{t+1}) = -((1 - A_1)\sigma_{g,t} - A_2 q\gamma)\sigma_\Lambda\rho. \tag{21}$$

Collecting like terms, we have:

$$g_t :\ (1 - A_1)\kappa + A_1 = 0 \tag{22}$$

$$\sigma_{b,t} :\ -A_2(\omega - 1) - (1 - A_1)A_2 q\gamma = (1 - A_1)\rho\sigma_\Lambda \tag{23}$$

$$\sigma_{g,t}^2 :\ (1 - A_1)\eta + \frac{1}{2}(1 - A_1)^2 = 0 \tag{24}$$

$$(1 - A_1)\bar{g} + \frac{1}{2}(A_2^2 q^2 + (1 - q^2)A_2^2)\gamma^2 = r_f + \rho\sigma_\Lambda A_2 q\gamma \tag{25}$$

Solving the above set of equations simultaneously implies that $A_1 = -\frac{\kappa}{1-\kappa}$, $A_2 = \frac{\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-\gamma q}$, and the "variance-in-mean" parameter is $\eta = -\frac{1}{2}\frac{1}{(1-\kappa)}$. To solve for A_0 we have $\bar{mb} = A_0 + A_1(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$ which implies $A_0 = \bar{mb} + \frac{\kappa}{1-\kappa}(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$. Using this and writing market values as stated in the text gives,

$$m_t = b_t + \alpha_0 + \alpha_1 g_t - \alpha_2\sigma_{g,t}, \tag{26}$$

where $\alpha_0 = -\overline{bm} - \alpha_1(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$, $\alpha_1 = \frac{\kappa}{1-\kappa}$, $\alpha_2 = \frac{\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-\gamma q}$ and $roe_t \equiv g_t$ since the firm does not pay dividends.

B. Derivation of Stock Returns

Combining the market value equation (26) with $E_t[r_{t+1}]+\frac{1}{2}V_t[r_{t+1}] = r_f - cov_t(\lambda_{t+1}, r_{t+1})$ allows us to calculate market returns as

$$r_{t+1} = E_t[r_{t+1}] - (r_{t+1} - E_t[r_{t+1}]), \tag{27}$$

$$= r_f - ((1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma)\rho + \frac{1}{2}\sigma_t^2$$

$$+ (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1} - \gamma\alpha_2 z_{t+1}. \tag{28}$$

which is the equation in the text.

C. Stock Return Variance

The conditional variance of the stock return is given by $V_t = E_t[(r_{t+1} - E_t[r_{t+1}])^2] = \sigma_t^2$. From (6) this implies

$$\sigma_t^2 = ((1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma)^2 + (1-q^2)\alpha_2^2\gamma^2. \tag{29}$$

D. Expected rates of equity returns

By the no arbitrage condition we have that $\mu_t = \log(E_t[e^{r_{t+1}}]) = E_t[r_{t+1}] + \frac{1}{2}V_t[r_{t+1}] = r_f - cov_t(\lambda_{t+1}, r_{t+1})$. From (6) we arrive at

$$\mu_t = r_f + \rho\sigma_\Lambda[(1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma]. \tag{30}$$

To express μ_t in terms of accounting-based variables we can use (26) to write the volatility of book growth as $\sigma_{g,t} = \frac{1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t]$. Substituting this into (30) we obtain

$$\mu_t = r_f + \rho\sigma_\Lambda[(1+\alpha_1)\frac{1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t] - \alpha_2 q\gamma].$$

This can be expressed as follows:

$$\mu_t = r_f + \theta_0 + \theta_1 bm_t + \theta_2 g_t, \tag{31}$$

where $\theta_0 = \theta_1(\alpha_0 - \alpha_2 q\gamma)$, $\theta_1 = (1-\omega) - \frac{\gamma q}{(1-\kappa)}$, $\theta_2 = \alpha_1\theta_1$and $roe_t \equiv g_t$ since the firm does not pay dividends.

E. Variance risk premiums

The price of a variance contract is its discounted payoff, and for no-arbitrage, it must satisfy the standard condition:

$$v_{t,t+1} = E_t[e^{\lambda_{t+1}}\sigma^2_{t+1}] = e^{-r_f}E_t[\sigma^2_{t+1}] + cov_t(e^{\lambda_{t+1}}, \sigma^2_{t+1}). \tag{32}$$

This implies that the variance risk premium is given by

$$E_t[\sigma^2_{t+1}] - e^{r_f}v_{t,t+1} = -e^{r_f}cov_t(e^{\lambda_{t+1}}, \sigma^2_{t+1}). \tag{33}$$

From (29) we have:

$$\sigma^2_t = ((1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma)^2 + (1-q^2)\alpha_2^2\gamma^2 \tag{34}$$

$$= (1+\alpha_1)^2\sigma^2_{g,t} - 2\omega(1+\alpha_1)\alpha_2\sigma_{g,t}\gamma$$

$$+ \alpha_2^2 q^2\gamma^2 + (1-q^2)\alpha_2^2\gamma^2. \tag{35}$$

Thus, next period expected variance is given by:

$$\sigma_{t+1}^2 = (1+\alpha_1)^2\sigma_{g,t+1}^2 - 2\omega\gamma(1+\alpha_1)\alpha_2\sigma_{g,t+1} + \alpha_2^2\gamma^2, \tag{36}$$

and

$$E_t[\sigma_{t+1}^2] = (1+\alpha_1)^2 E_t[\sigma_{g,t+1}^2] - 2\omega\gamma(1+\alpha_1)\alpha_2 E_t[\sigma_{g,t+1}] + \alpha_2^2\gamma^2, \tag{37}$$

$$= (1+\alpha_1)^2(\omega^2\sigma_{g,t}^2 + \gamma^2)$$

$$- 2\omega^2(1+\alpha_1)\alpha_2\gamma\sigma_{g,t} + \alpha_2^2\gamma^2. \tag{38}$$

We need to determine the covariance term,

$$cov_t(e^{\lambda_{t+1}}, \sigma_{t+1}^2) = (1+\alpha_1)^2 cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}^2)$$

$$+ -2\omega\gamma(1+\alpha_1)\alpha_2 cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}). \tag{39}$$

In order to solve this we need to determine $cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}^2)$ and $cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1})$, where $\sigma_{g,t+1}^2 = (\omega\sigma_{g,t} + \gamma z_{t+1})^2$ and $\sigma_{g,t+1} = \omega\sigma_{g,t} + \gamma z_{t+1}$. Given that λ_{t+1} and $z_{t+1} = q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}$are normal, we have

$$cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}) = \gamma cov_t(e^{\lambda_{t+1}}, q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}), \tag{40}$$

$$= -\gamma e^{-r_f} q\rho\sigma_\Lambda. \tag{41}$$

To calculate the second covariance term, note that $(\omega\sigma_{g,t} + \gamma z_{t+1})^2 = \omega\sigma_{g,t}^2 + 2\omega\gamma z_{t+1}\sigma_{g,t} + \gamma^2 z_{t+1}^2$. Thus,

$$cov(e^{\lambda_{t+1}}, \sigma^2_{g,t+1}) = -e^{-r_f}\rho q \sigma_\Lambda 2\omega\gamma\sigma_{g,t} + \gamma^2 cov(e^{\lambda_{t+1}}, \gamma z^2_{t+1}). \tag{42}$$

To solve for the second term, we have

$$\gamma^2 cov(e^{\lambda_{t+1}}, z^2_{t+1}) = \gamma^2 E_t((e^{\lambda_{t+1}} - e^{-r})(q^2\epsilon^2_{t+1} + 2q\sqrt{1-q^2}\epsilon_{t+1}\xi_{t+1} + (1-q^2)\xi^2_{t+1})), \tag{43}$$

$$= \gamma^2 E_t((e^{\lambda_{t+1}} - e^{-r})(q^2\epsilon^2_{t+1}). \tag{44}$$

Since ϵ_{t+1} is normal, we can decompose it into $\epsilon_{t+1} = \rho w_{t+1} + \sqrt{1-\rho^2}w^*_{t+1}$ where w^*_{t+1} is an independent normal distribution. This implies

$$\gamma^2 E_t((e^{\lambda_{t+1}} - e^{-r})(q^2\epsilon^2_{t+1}) = -\gamma^2(e^{-r_f}q^2 + q^2 E_t[e^{\lambda_{t+}}\epsilon^2_{t+1}]) \tag{45}$$

$$= -\gamma^2 e^{-r_f} q^2(1 + \rho^2(\sigma^2_\Lambda + 1)). \tag{46}$$

Thus

$$cov_t(e^{\lambda_{t+1}}, \sigma^2_{g,t+1}) = -e^{-r_f}\rho\sigma_\Lambda 2\omega\gamma\sigma_{g,t} - \gamma^2 e^{-r_f} q^2(1 + \rho^2(\sigma^2_\Lambda + 1)). \tag{47}$$

Plugging this back into (42), we obtain

$$\begin{aligned} e^{r_f} cov_t(e^{\lambda_{t+1}}, \sigma^2_{t+1}) &= -(1+\alpha_1)^2(\rho\sigma_\Lambda 2\omega\gamma\sigma_{g,t} + \gamma^2 q^2(1 + \rho^2(\sigma^2_\Lambda + 1))) \\ &\quad + 2\omega^2(1+\alpha_1)\alpha_2\gamma q\rho\sigma_\Lambda, \qquad (48) \\ &= -2(1+\alpha_1)^2\rho\sigma_\Lambda\omega\gamma\sigma_{g,t} \end{aligned}$$

$$- (1+\alpha_1)^2\gamma^2 q^2(1+\rho^2(\sigma_\Lambda^2+1)) + 2\omega^2(1+\alpha_1)\alpha_2\gamma q\rho\sigma_\Lambda, \quad (49)$$

$$= \eta_1\sigma_{g,t} + \eta_0. \quad (50)$$

where $\eta_0 = [-(1+\alpha_1)\gamma q(1+\rho^2(\sigma_\Lambda^2+1))+2\omega^2\alpha_2\rho\sigma_\Lambda]q\gamma(1+\alpha_1)$ and $\eta_1 = -(1+\alpha_1)^2 q\rho\sigma_\Lambda 2\omega\gamma$.

Thus, the variance risk premium is given by

$$E_t[\sigma_{t+1}^2] - e^{r_f}v_t = \eta_0 + \eta_1\sigma_{g,t}. \quad (51)$$

But we can use the fact that $\sigma_{g,t} = \frac{1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t]$ to obtain

$$E_t[\sigma_{t+1}^2] - e^{-r_f}v_t = -\eta_0 + -\frac{\eta_1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t] \quad (52)$$

But $\frac{\eta_1}{\alpha_2} = \frac{-(1+\alpha_1)^2 q\rho\sigma_\Lambda 2\omega\gamma}{\frac{-\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-\gamma q}} = q(1+\alpha_1)^2\omega\gamma((1-\kappa)(1-\omega)-\gamma q) = \phi_1$ which is negative if $q < 0$. Thus

$$E_t[\sigma_{t+1}^2] - e^{-r_f}v_t = \phi_0 + \phi_1 bm_t + \phi_2 g_t, \quad (53)$$

where $\phi_0 = \alpha_0\phi_1 + \eta_0$, $\phi_1 = q(1+\alpha_1)^2\omega\gamma((1-\kappa)(1-\omega)-\gamma q)$, $\phi_2 = \alpha_1\phi_1$, $\eta_0 = -[(1+\alpha_1)\gamma q(1+\rho^2(\sigma_\Lambda^2+1)) - 2\omega^2\alpha_2\rho\sigma_\Lambda]q\gamma(1+\alpha_1)$, and $roe_t \equiv g_t$ since the firm does not pay dividends. This expression delivers the equation in the main body of the text.

F. A relation between book growth and market volatility

Our model of book growth depends upon $\sigma_{g,t}^2$, which is difficult to estimate due to limitations in times series data. Given that, $g_{t+1} = \bar{g} + \kappa g_t + \eta\sigma_{g,t}^2 + \sigma_{g,t}\epsilon_{t+1}$, where $\eta = -\frac{1}{2}\frac{1}{(1-\kappa)}$, we can substitute out $\sigma_{g,t}^2$ by using (34). Combining this with (26) implies that future book growth can be written as a function of current growth, expected market variance and bm_t:

$$g_{t+1} = \varphi_0 + \varphi_1 g_t + \varphi_2 bm_t + \varphi_3 \sigma_t^2 + \sigma_{g,t}\epsilon_{t+1}, \tag{54}$$

where $\varphi_0 = \bar{g} + \omega\gamma\alpha_0 + \frac{1}{2}\frac{1}{1+\alpha_1}\alpha_2^2\gamma^2$, $\varphi_1 = (1 - \frac{\omega\gamma}{1-\kappa})\kappa$, $\varphi_2 = -\omega\gamma$, and $\varphi_3 = -\frac{1}{2}(1+\kappa)$. The α_0, α_1, and α_2 terms are defined above.

G. Measuring the price of a variance contract

We want the price of a variance contract from time t to some future date τ, $v_{t,t+\tau} = E_t[\frac{\Lambda_{t+\tau}}{\Lambda_t}\sum_{i=1}^{\tau} E_t([r_{t+i} - E_t[r_{t+i}])^2]$. To recover this value, we can use the market price of a contract that pays off the logarithm of the stock price. From (6) we have,

$$m_{t+1} = m_t + \mu_t - \frac{1}{2}\sigma_t^2 + (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1} - \alpha_2\gamma z_{t+1}, \tag{55}$$

$$= m_t + \mu_t - \frac{1}{2}\sigma_t^2 + \eta_{t+1}. \tag{56}$$

The time $t+\tau$ log price is then given by

$$m_{t+\tau} = m_t + \sum_{i=1}^{\tau}\mu_{t+i-1} - \frac{1}{2}\sum_{i=1}^{\tau}\sigma_{t+i-1}^2 + \sum_{i=1}^{\tau}\eta_{t+i}. \tag{57}$$

The price of the log-contract is then given by

$$f_{t,t+\tau} = E_t[\frac{\Lambda_{t+\tau}}{\Lambda_T} m_{t+\tau}], \tag{58}$$

$$= e^{-r_f(t+\tau)}(m_t + r_f(t+\tau)) - \frac{1}{2}E_t[\frac{\Lambda_{t+\tau}}{\Lambda_t}\sum_{i=1}^{\tau}\sigma_{t+i-1}^2]. \tag{59}$$

This implies that the price of a contract that pays the cumulative variance from time t to $t+\tau$ is $v_{t,t+\tau} = 2(e^{-r_f(t+\tau)}(m_t + r_f(t+\tau)) - f_{t,t+\tau})$.

H. The price of the log contract

We apply the model free equation provided by Bakshi and Madan (2000) where any twice differentiable function $F(S)$ can be expressed as:

$$F(S) = F(\bar{S}) + (S - \bar{S})F_S(\bar{S}) \tag{60}$$

$$+ \int_{\bar{S}}^{\infty} F_{SS}(K)(S-K)^+ dK + \int_0^{\bar{S}} F_{SS}(K)(K-S)^+ dK, \tag{61}$$

where $\bar{S}$ is an arbitrary real constant.

Let $F(S) = \log(M_{t+\tau}) = m_{t+\tau}$, then

$$m_{t+\tau} = \log(\bar{S}) + \frac{(M_{t+\tau} - \bar{S})}{\bar{S}} - \int_{\bar{S}}^{\infty} \frac{(M_{t+\tau} - K)^+ dK}{K^2} - \int_0^{\bar{S}} \frac{(K - M_{t+\tau})^+ dK}{K^2}. \tag{62}$$

The value of a log contract is thus

$$\begin{aligned} f_{t,t+\tau} &= E^Q[e^{-r_{t+\tau}} m_{t+\tau}] = e^{-r_{t+\tau}} \log(F_{t+\tau}) \\ &- \int_{F_{t+\tau}}^{\infty} \frac{C(K, t+\tau)dK}{K^2} - \int_0^{F_{t+\tau}} \frac{P(K, t+\tau)dK}{K^2}. \end{aligned} \tag{63}$$

where $F_{t,t+\tau}$ is a forward contract on the equity, while $C(K, t+\tau)$ and $P(K, t+\tau)$ represent call and put contracts respectively. This implies that the price of variance can be given by,

$$\begin{aligned} v_{t,t+\tau} &= 2(e^{-r_f(t+\tau)}(m_t + r_f(t+\tau)) - e^{-r_{t+\tau}} \log(F_{t+\tau}) \\ &+ \int_{F_{t+\tau}}^{\infty} \frac{C(K, t+\tau)dK}{K^2} + \int_0^{F_{t+\tau}} \frac{P(K, t+\tau)dK}{K^2}). \end{aligned} \tag{64}$$

We approximate this equation using OptionMetrics' volatility surface files along with their estimate of the forward contract, $F_{t+\tau}$.

B. Figures

Figure I: Time Series Plot of Median Book-to-Market, ROE, and Variance Returns

Figure I plots the monthly median excess return on a variance contract (in percentage) of a 30 day ahead variance contract, $R^v_{t,t+1} - R_f$, as well as median book-to-market, $bm_t = \log(\frac{B_t}{M_t})$, and median return on equity, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$. All accounting variables are based on quarterly financial reports from 1996 to 2013. All series have been smoothed using a five point moving average.

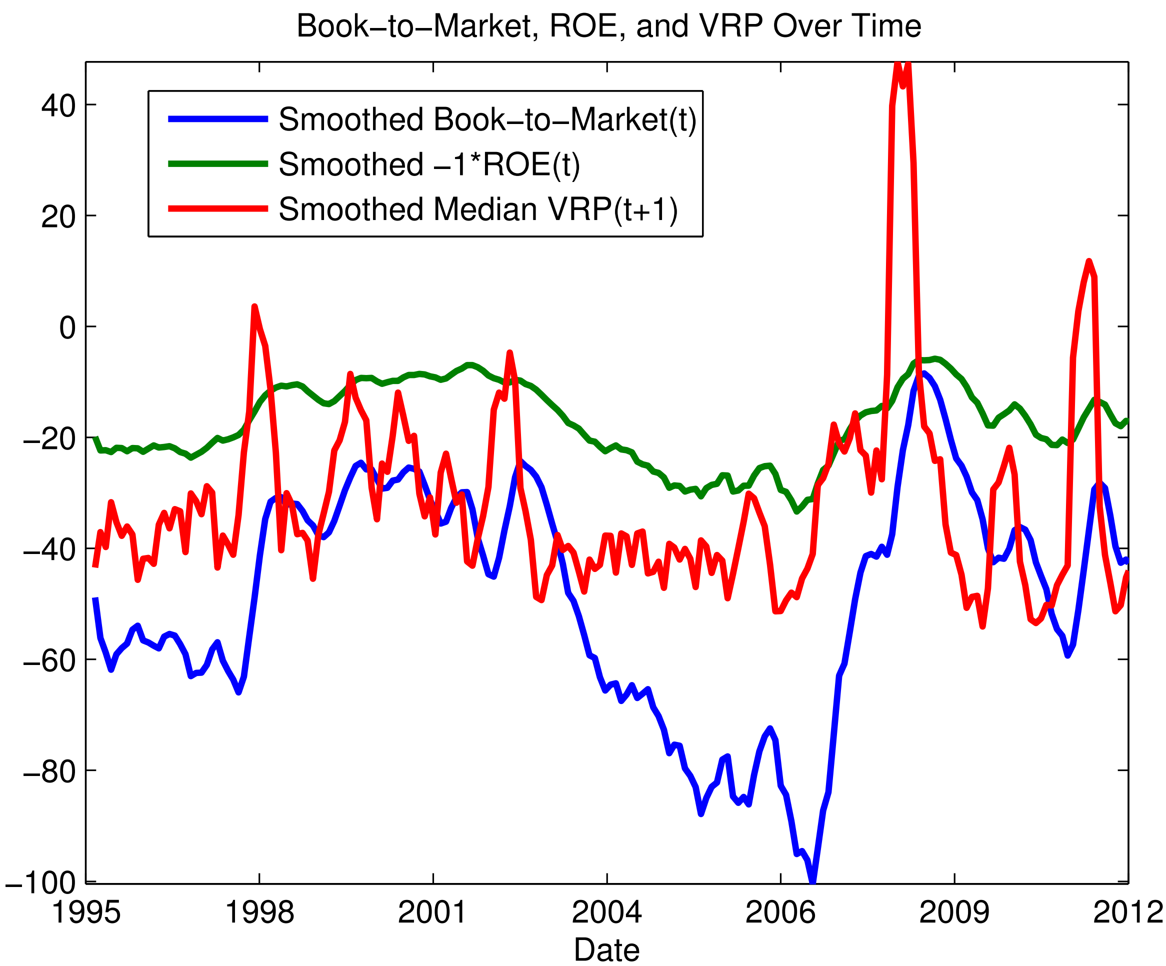
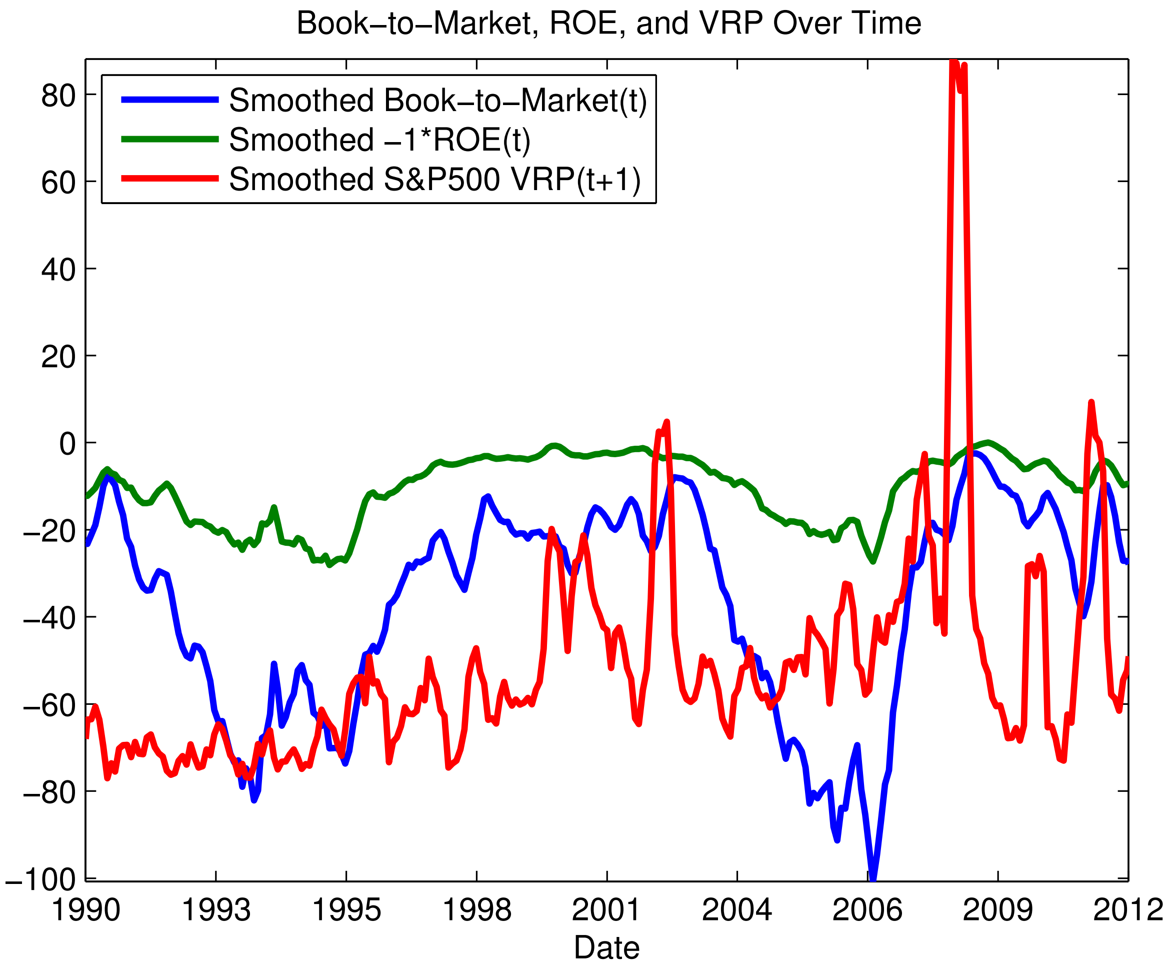


Figure II: Time Series Plot of Book-to-Market, ROE, and S&P 500 Variance Returns

Figure II plots the monthly median excess return on a variance contract (in percentage) of a 30 day ahead S&P 500 variance contract based on the data used in Bollerslev et al. (2009), as well as median book-to-market, $bm_t = \log(\frac{B_t}{M_t})$, and median return on equity, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$. All accounting variables are based on quarterly financial reports from 1990 to 2013. All series have been smoothed using a five point moving average.



C. Tables

Table I: Summary Statistics

Table I reports summary statistics of key variables used in the analysis and other common firm-level characteristics. $v_{t,t+1}$ represents the price of a 30 day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. σ^2_{t+1} represents the 30 day ahead realized variance, which is calculated using the sum of squared daily log returns. R_f is the gross risk free rate obtained from OptionMetrics zero coupon bond file. $R^v_{t,t+1} - R_f$ is the excess return (in percentage) on a 30 day ahead variance contract. R_{t+1} is the 30 day ahead net stock return (in percentage). $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio. $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity. $size_t = log(M_t)$ is the log of market capitalization. β is a firm's rolling 5 year (60 month) historical "beta" estimated using the market model, and $lvar$ is lagged 30 day variance.

	Mean	Std Dev	P10	P25	P50	P75	P90
$100\times v_{t,t+1}$	2.79	3.06	0.63	1.08	1.92	3.39	5.83
$100\times\sigma^2_{t+1}$	2.29	3.62	0.27	0.52	1.13	2.54	5.34
$100\times(\sigma^2_{t+1} - v_{t,t+1}R_f)$	-0.44	2.87	-2.58	-1.32	-0.50	0.02	1.32
$R^v_{t,t+1} - R_f$	-16.72	77.95	-78.20	-62.24	-37.24	1.23	61.88
$R_{t,t+1}$	0.77	14.98	-14.99	-6.47	0.52	7.36	16.03
bm_t	-0.89	0.77	-1.90	-1.35	-0.84	-0.38	0.01
$100\times roe_t$	1.38	9.14	-5.63	0.39	2.75	4.78	7.49
$size_t$	7.28	1.57	5.33	6.13	7.15	8.26	9.45
β	1.31	0.85	0.42	0.73	1.15	1.72	2.41
$100\times lvar_{t-1,t}$	2.40	3.50	0.31	0.60	1.26	2.74	5.54

Table II: Correlation Matrix

Table II reports the correlation matrix of key variables used in the analysis and other common firm-level characteristics. $v_{t,t+1}$ represents the price of a 30 day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. σ^2_{t+1} represents the 30 day ahead realized variance, which is calculated using the sum of squared daily log returns. R_f is the gross risk free rate obtained from OptionMetrics zero coupon bond file. $R^v_{t+1} - R_f$ is the excess return (in percentage) on a 30 day ahead variance contract. $R_{t,t+1}$ is the 30 day ahead net stock return (in percentage). $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio. $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity. $size_t = log(M_t)$ is the log of market capitalization. β is a firm's rolling 5 year (60 month) historical "beta" estimated using the market model, and $lvar_{t-1,t}$ is lagged monthly variance estimated from daily stock return data.

$100\times v_{t,t+1}$		0.515	-0.361	-0.059	-0.025	0.030	-0.355	-0.463	0.363	0.542
$100\times\sigma^2_{t+1}$	0.515		0.492	0.658	-0.029	-0.011	-0.257	-0.366	0.368	0.499
$100\times(\sigma^2_{t+1} - v_{t,t+1}R_f)$	-0.361	0.492		0.767	-0.023	-0.049	0.111	0.144	-0.001	-0.065
$R^v_{t,t+1} - R_f$	-0.059	0.658	0.767		-0.025	-0.049	-0.008	-0.021	0.116	0.103
$R_{t,t+1}$	-0.025	-0.029	-0.023	-0.025		0.010	0.017	0.009	-0.012	-0.029
bm_t	0.030	-0.011	-0.049	-0.049	0.010		-0.083	-0.234	-0.026	-0.008
$100\times roe_t$	-0.355	-0.257	0.111	-0.008	0.017	-0.083		0.302	-0.205	-0.285
$size_t$	-0.463	-0.366	0.144	-0.021	0.009	-0.234	0.302		-0.272	-0.404
β	0.363	0.368	-0.001	0.116	-0.012	-0.026	-0.205	-0.272		0.406
$100\times lvar_{t-1,t}$	0.542	0.499	-0.065	0.103	-0.029	-0.008	-0.285	-0.404	0.406	

Table III: Cross-Sectional Regressions

Table III reports mean coefficients and t-statistics based on the Fama and MacBeth (1973) approach (with a Newey-West correction) of one-period-ahead excess variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity (both variables are deflated by the price of a variance contact $v_{t,t+1}$ as per equation (11)). $v_{t,t+1}$ is the price of a 30 (60 in column 6) day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
bm_t	-0.056***			-0.077***	-0.055***	-0.114**
	(-6.026)			(-8.071)	(-5.451)	(-2.590)
roe_t		0.312***		-0.511***	-0.609***	-1.600***
		(2.885)		(-6.113)	(-8.855)	(-7.616)
$\frac{1}{v_{t,t+1}}$			0.073***		0.054**	0.091
			(3.712)		(2.269)	(1.359)
Intercept	-22.727***	-19.396***	-23.955***	-22.913***	-24.955***	-19.482***
	(-10.234)	(-8.382)	(-12.111)	(-10.325)	(-12.894)	(-4.689)
# Observations	307,510	307,510	307,510	307,510	307,510	307,510
R^2	0.012	0.005	0.014	0.015	0.022	0.023

Table IV: Cross-Sectional Regressions with Additional Controls

Table IV reports mean coefficients and t-statistics based on the Fama and MacBeth (1973) approach (with a Newey-West correction) of one-period-ahead excess variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity (both variables are deflated by the price of a variance contact $v_{t,t+1}$ as per equation (11)). $v_{t,t+1}$ is the price of a 30 (60 in columns 4 though 6) day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. $R_{t-1,t}$, $R_{t-12,t-1}$, $R_{t-36,t-13}$ represents lagged monthly returns, lagged yearly returns excluding the most recent month, and lagged three year returns excluding the most recent annual return, respectively. $size_t = log(M_t)$ is the log of market capitalization, $lvar_{t-1,t}$ is lagged monthly variance estimated from daily stock return data, and $Jump_t$ is a jump risk proxy which is the difference between call and put option implied volatility (Yan, 2011). *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
		←30 day→			←60 day→	
bm_t	-0.055***	-0.061***	-0.062***	-0.111***	-0.145***	-0.143***
	(-3.474)	(-3.788)	(-4.068)	(-2.616)	(-3.300)	(-3.376)
roe_t	-0.576***	-0.468***	-0.468***	-1.518***	-1.225***	-1.208***
	(-6.708)	(-5.315)	(-5.339)	(-7.798)	(-5.918)	(-5.949)
$\frac{1}{v_{t,t+1}}$	0.083***	0.149***	0.166***	0.160**	0.325***	0.361***
	(2.694)	(4.369)	(4.322)	(2.217)	(4.219)	(4.144)
$R_{t-1,t}$	-0.058		-0.103**	-0.011		-0.073
	(-1.075)		(-2.169)	(-0.162)		(-1.200)
$R_{t-12,t-1}$	-0.736**		-0.672**	-0.463*		-0.413*
	(-2.381)		(-2.380)	(-1.709)		(-1.698)
$R_{t-36,t-13}$	3.480***		1.648**	3.196***		1.111**
	(5.232)		(2.587)	(5.542)		(2.211)
$size_t$		-2.294***	-2.276***		-3.867***	-3.836***
		(-5.599)	(-5.932)		(-5.612)	(-5.893)
$lvar_{t-1,t}$		0.340***	0.310***		0.360***	0.331***
		(15.939)	(15.003)		(12.366)	(11.481)
$Jump_t$		0.185***	0.171***		0.277***	0.271***
		(4.457)	(3.936)		(4.222)	(4.029)
Intercept	-28.568***	-21.210***	-23.341***	-23.570***	-6.640	-8.974**
	(-11.527)	(-6.810)	(-7.772)	(-6.103)	(-1.516)	(-2.139)
# Observations	307,510	307,510	307,510	307,510	307,510	307,510
R^2	0.033	0.051	0.058	0.036	0.063	0.071

Table V: Cross-Sectional Regressions with Controls for S&P 500 firms

Table V reports mean coefficients and t-statistics based on the Fama and MacBeth (1973) approach (with a Newey-West correction) of one-period-ahead excess variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity (both variables are deflated by the price of a variance contact $v_{t,t+1}$ as per equation (11)). $v_{t,t+1}$ is the price of a 30 (60 in columns 4 though 6) day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. $R_{t-1,t}$, $R_{t-12,t-1}$, $R_{t-36,t-13}$ represents lagged monthly returns, lagged yearly returns excluding the most recent month, and lagged three year returns excluding the most recent annual return, respectively. $size_t = log(M_t)$ is the log of market capitalization, $lvar_{t-1,t}$ is lagged monthly variance estimated from daily stock return data, and $Jump_{t-1,t}$ is a jump risk proxy which is the difference between call and put option implied volatility (Yan, 2011). *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2) ←30 day→	(3)	(4)	(5) ←60 day→	(6)
bm_t	-0.035***	-0.022**	-0.024***	-0.078**	-0.057**	-0.055**
	(-3.113)	(-2.384)	(-2.664)	(-2.581)	(-2.315)	(-2.296)
roe_t	-0.476***	-0.408***	-0.393***	-1.482***	-1.248***	-1.175***
	(-5.765)	(-5.014)	(-4.919)	(-6.066)	(-5.392)	(-5.408)
$\frac{1}{v_{t,t+1}}$	0.160***	0.232***	0.245***	0.322***	0.495***	0.523***
	(6.177)	(7.826)	(7.393)	(6.183)	(7.634)	(7.290)
$R_{t-1,t}$	-0.203**		-0.233***	-0.125		-0.163*
	(-2.433)		(-3.317)	(-1.062)		(-1.699)
$R_{t-12,t-1}$	1.161*		0.818	2.305***		1.688***
	(1.850)		(1.283)	(3.769)		(2.885)
$R_{t-36,t-13}$	5.827***		3.301**	6.760***		3.912***
	(4.481)		(2.544)	(7.026)		(4.442)
$size_t$		-0.575	-0.519		-1.296*	-1.286*
		(-1.097)	(-1.006)		(-1.655)	(-1.740)
$lvar_{t-1,t}$		1.375***	1.313***		1.564***	1.483***
		(14.835)	(14.354)		(12.079)	(11.712)
$Jump_{t-1,t}$		0.162	0.117		0.583***	0.546***
		(1.112)	(0.818)		(3.171)	(3.002)
Intercept	-42.507***	-52.928***	-55.945***	-42.462***	-48.527***	-50.829***
	(-15.386)	(-11.159)	(-11.593)	(-8.576)	(-8.696)	(-9.199)
# Observations	79,017	79,017	79,017	79,017	79,017	79,017
R^2	0.090	0.129	0.143	0.104	0.164	0.180

Table VI: Time Series regressions

Table VI reports the results of time series regressions of excess 30 day ahead variance returns on the variables show. $bm_t = \log(\frac{B_t}{M_t})$ represent the median book-to-market ratio and $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ represent median return on equity (both variables are deflated by the price of a variance contact $v_{t,t+1}$ as per equation (11)). $v_{t,t+1}$ in columns 1 through 5 represents the median price of a 30 day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. $v_{t,t+1}$ in columns 6 through 10 represents a 30 day ahead S&P 500 variance contract based on the data used in Bollerslev et al. (2009). t-statistics are calculated from robust standard errors corrected for heteroskedasticity. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			Variance Returns					S&P 500 Index Variance Returns		
bm_t	0.002***			-0.003**	-0.006***	0.001			-0.007***	-0.007**
	(2.677)			(-2.091)	(-3.542)	(1.179)			(-5.381)	(-2.454)
roe_t		-0.082***		-0.186***	-0.088*		-0.084***		-0.287***	-0.286***
		(-3.387)		(-3.448)	(-1.658)		(-3.283)		(-7.541)	(-3.735)
$\frac{1}{v_{t,t+1}}$			-0.003***		-0.006***			-0.001*		-0.000
			(-3.758)		(-3.573)			(-1.933)		(-0.010)
Intercept	-0.157***	-0.118**	-0.076	-0.114**	-0.047	-0.473***	-0.422***	-0.440***	-0.473***	-0.473***
	(-3.164)	(-2.253)	(-1.294)	(-2.151)	(-0.774)	(-9.393)	(-9.144)	(-8.041)	(-9.625)	(-7.725)
# Observations	215	215	215	215	215	287	287	287	287	287
R^2	0.033	0.052	0.071	0.063	0.100	0.004	0.030	0.012	0.070	0.070

Table VII: Cross-Sectional Regressions with Forward Controls

Table VII reports mean coefficients and t-statistics based on the Fama and MacBeth (1973) approach (with a Newey-West correction) of one-period-ahead excess variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity (both variables are deflated by the price of a variance contact $v_{t,t+1}$ as per equation (11)). Columns 1 and 5 include full sample firm specific slopes based on the Fama and French (1993) three factor asset pricing model. Columns 2 and 6 include full sample firm specific slopes based on the Fama and French (2015) five factor asset pricing model. Columns 3 and 7 include full sample firm specific slopes based on the Fama and French (2015) five factor asset pricing model plus an additional factor, which represents the percentage change in the VIX. Columns 4 and 8 include full sample firm specific slopes based on the Fama and French (2015) five factor asset pricing model plus an additional factor which represents changes in the VIX plus all lagged control variables listed in Table IV. $R_{t,t+1}$ represents one-period ahead stock returns . *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			←30 day→				←60 day→		
	bm_t	-0.084***	-0.083***	-0.083***	-0.061***	-0.175***	-0.175***	-0.170***	-0.155***
		(-7.167)	(-7.248)	(-7.222)	(-4.860)	(-6.057)	(-6.161)	(-6.041)	(-5.239)
	roe_t	-0.306***	-0.295***	-0.314***	-0.363***	-0.969***	-0.987***	-1.033***	-0.951***
		(-4.312)	(-4.109)	(-4.746)	(-5.397)	(-4.751)	(-5.049)	(-6.288)	(-5.842)
Forward Controls	$R_{t,t+\tau}$	-0.318***	-0.303***	-0.292***	-0.302***	-0.373***	-0.353***	-0.332***	-0.337***
		(-3.023)	(-2.963)	(-2.911)	(-2.904)	(-3.885)	(-3.925)	(-3.839)	(-3.798)
	β_m	-0.013***	-0.012**	-0.016***	-0.015***	-0.029**	-0.033***	-0.032**	-0.036***
		(-2.671)	(-2.593)	(-3.224)	(-3.210)	(-2.466)	(-2.704)	(-2.471)	(-2.791)
	β_{hml}	0.413**	0.250*	0.282**	0.293**	0.374	0.066	0.241	0.260
		(2.190)	(1.705)	(1.979)	(2.191)	(0.822)	(0.169)	(0.704)	(0.802)
	β_{smb}	-0.706***	-0.518***	-0.509***	-0.468***	-1.343***	-1.196***	-1.205***	-0.977***
		(-4.292)	(-3.536)	(-3.449)	(-3.452)	(-3.783)	(-3.649)	(-3.819)	(-3.230)
	β_{rmw}		0.466***	0.448***	0.379***		0.688**	0.801**	0.679**
			(3.212)	(3.176)	(2.793)		(2.182)	(2.435)	(2.102)
	β_{cma}		0.198**	0.186*	0.172*		0.190	0.244	0.257
			(2.043)	(1.953)	(1.860)		(0.994)	(1.168)	(1.248)
	β_{vix}			3.862***	4.113***			4.496	6.278**
				(2.827)	(3.614)			(1.464)	(2.150)
Lagged Controls		NO	NO	NO	YES	NO	NO	NO	YES
	Intercept	-31.087***	-31.419***	-32.497***	-0.302***	-29.016***	-29.894***	-30.034***	-14.715***
		(-14.223)	(-15.153)	(-17.297)	(-2.904)	(-11.705)	(-14.271)	(-14.967)	(-3.927)
	# Observations	307,510	307,510	307,510	307,510	307,510	307,510	307,510	307,510
	R^2	0.073	0.085	0.093	0.129	0.088	0.105	0.117	0.160

Table VIII: Variance Return Portfolio Sorts

Table VIII reports portfolio returns based on two-way sorts of bm_t and roe_t. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity. Firms are first sorted on bm_t and then on roe_t. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively. The (V-I) column reports the return of a high *roe* minus a low *roe* portfolio. α_{FF3}, α_{FF5}, and $\alpha_{FF5+VIX}$ represent the time series intercepts of the the return of a high *roe* minus a low *roe* portfolio based on the Fama and French (1993) three factor model, the Fama and French (2015) five factor model, and a Fama and French (2015) five factor plus an additional factor, which represents the percentage change in the VIX, respectively. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively. Significance levels are based on standard errors corrected for heteroskedasticity.

(a) Panel A: 30 Day Ahead

		$roe(\rightarrow)$							
		I (Low)	II	III	IV	V (Hi)	(V-I)	α_{FF3}	α_{FF5}	$\alpha_{FF5+VIX}$
	1 (low)	-16.18	-7.23	-9.04	-10.63	-15.61	0.60	-1.34	-1.25	-0.83
	2	-14.32	-11.71	-14.22	-16.86	-15.37	-1.05	-2.89*	-2.98*	-2.03
$bm(\downarrow)$	3	-14.97	-16.34	-18.82	-21.89	-18.47	-3.52***	-3.09***	-3.00***	-2.38*
	4	-18.00	-21.86	-25.20	-25.38	-22.62	-4.62***	-4.99***	-4.95***	-4.51***
	5 (High)	-17.40	-22.83	-24.89	-26.69	-24.06	-6.63***	-7.08***	-7.00***	-6.57***

(b) Panel B: 60 Day Ahead

				$roe(\rightarrow)$						
		I (Low)	II	III	IV	V (Hi)	(V-I)	α_{FF3}	α_{FF5}	$\alpha_{FF5+VIX}$
	1 (low)	-10.16	-0.01	-3.26	-7.20	-12.18	-2.02	-5.49***	-5.05***	-4.35***
	2	-7.64	-5.31	-8.87	-12.47	-11.55	-3.90***	-5.05***	-5.33***	-4.17***
$bm(\downarrow)$	3	-9.34	-10.22	-13.74	-18.08	-14.68	-5.36***	-4.10***	-4.13***	-3.59***
	4	-12.46	-16.10	-20.44	-21.49	-18.47	-5.99***	-6.11***	-6.21***	-5.51***
	5 (High)	-11.73	-17.93	-18.67	-20.96	-19.12	-7.36***	-7.73***	-7.47***	-6.79***

Table IX: Cross-Sectional Regressions using QGARCH Realized Variance

Table IX reports mean coefficients and t-statistics based on the Fama and MacBeth (1973) approach (with a Newey-West correction) of one-period-ahead excess variance returns, calculated based on cumulative realized variance estimated using a QGARCH(1,1) model, on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity (both variables are deflated by the price of a variance contact $v_{t,t+1}$ as per equation (11)). $v_{t,t+1}$ is the price of a 30 (60 in columns 4 though 6) day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. $R_{t-1,t}$, $R_{t-12,t-1}$, $R_{t-36,t-13}$ represents lagged monthly returns, lagged yearly returns excluding the most recent month, and lagged three year returns excluding the most recent annual return, respectively. $size_t = log(M_t)$ is the log of market capitalization, $lvar_{t-1,t}$ is lagged monthly variance estimated from daily stock return data, and $Jump_t$ is a jump risk proxy which is the difference between call and put option implied volatility (Yan, 2011). Forward controls include all variables included as forward control variables listed in columns 4 and 8 of Table IV. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		←30 day→				←60 day→		
bm_t	-0.087***	-0.094***	-0.095***	-0.094***	-0.171***	-0.221***	-0.219***	-0.222***
	(-8.547)	(-10.780)	(-11.131)	(-12.737)	(-6.481)	(-8.683)	(-10.606)	(-12.959)
roe_t	-0.995***	-0.707***	-0.704***	-0.651***	-2.387***	-1.726***	-1.704***	-1.570***
	(-15.410)	(-13.612)	(-13.571)	(-14.119)	(-17.054)	(-12.983)	(-13.817)	(-13.599)
$\frac{1}{v_{t,t+1}}$	0.107***	0.289***	0.301***	0.296***	0.225***	0.598***	0.628***	0.600***
	(5.355)	(9.445)	(9.217)	(10.281)	(4.675)	(9.131)	(9.682)	(11.640)
$R_{t-1,t}$	0.007		-0.159***	-0.153***	0.083		-0.079**	-0.048*
	(0.172)		(-6.035)	(-6.589)	(1.592)		(-2.316)	(-1.954)
$R_{t-12,t-1}$	0.386		0.252	0.426**	0.132		0.184	0.448**
	(1.327)		(1.147)	(2.110)	(0.422)		(0.765)	(2.061)
$R_{t-36,t-13}$	8.163***		3.551***	3.693***	8.087***		3.645***	3.947***
	(13.218)		(7.540)	(8.741)	(12.482)		(7.168)	(8.706)
$size_t$		-5.064***	-4.949***	-5.104***		-7.310***	-7.163***	-7.362***
		(-12.843)	(-12.996)	(-13.464)		(-11.049)	(-13.898)	(-13.502)
$lvar_{t-1,t}$		0.985***	0.974***	0.946***		0.824***	0.805***	0.742***
		(17.481)	(17.041)	(16.379)		(13.659)	(16.615)	(12.106)
$Jump_t$		0.201***	0.175***	0.222***		0.279***	0.261***	0.304***
		(4.692)	(4.118)	(5.538)		(4.212)	(4.666)	(4.882)
Forward Controls	NO	NO	NO	YES	NO	NO	NO	YES
Intercept	-21.543***	-12.845***	-15.217***	-17.835***	-18.250***	8.469*	5.713	2.391
	(-14.375)	(-3.893)	(-4.696)	(-5.380)	(-7.369)	(1.671)	(1.415)	(0.508)
# Observations	275,093	275,093	275,093	275,093	275,093	275,093	275,093	275,093
R^2	0.066	0.218	0.225	0.275	0.062	0.195	0.202	0.276

Table X: Cross-Sectional Regressions using QGARCH Expected Variance

Table X reports mean coefficients and t-statistics based on the Fama and MacBeth (1973) approach (with a Newey-West correction) of one-period-ahead expected excess variance returns, calculated based on expected cumulative variance using estimated parameters from a QGARCH(1,1) model, on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity (both variables are deflated by the price of a variance contact $v_{t,t+1}$ as per equation (11)). $v_{t,t+1}$ is the price of a 30 (60 in columns 4 though 6) day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. $R_{t-1,t}$, $R_{t-12,t-1}$, $R_{t-36,t-13}$ represents lagged monthly returns, lagged yearly returns excluding the most recent month, and lagged three year returns excluding the most recent annual return, respectively. $size_t = log(M_t)$ is the log of market capitalization, $lvar_{t-1,t}$ is lagged monthly variance estimated from daily stock return data, and $Jump_t$ is a jump risk proxy which is the difference between call and put option implied volatility (Yan, 2011). Forward controls include all variables included as forward control variables listed in columns 4 and 8 of Table IV. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		←30 day→				←60 day→		
bm_t	-0.129***	-0.140***	-0.140***	-0.141***	-0.306***	-0.379***	-0.374***	-0.365***
	(-15.152)	(-21.565)	(-21.369)	(-21.901)	(-12.228)	(-19.409)	(-19.574)	(-21.015)
roe_t	-1.495***	-1.054***	-1.044***	-1.045***	-4.225***	-2.945***	-2.913***	-2.909***
	(-15.475)	(-14.307)	(-14.262)	(-14.856)	(-12.219)	(-9.861)	(-9.751)	(-10.088)
$\frac{1}{v_{t,t+1}}$	0.121***	0.401***	0.408***	0.403***	0.398***	1.118***	1.134***	1.125***
	(8.046)	(13.967)	(13.916)	(14.332)	(7.928)	(14.684)	(14.630)	(14.794)
$R_{t-1,t}$	0.176***		-0.098***	-0.098***	0.433***		0.082**	0.084**
	(3.441)		(-3.704)	(-3.745)	(5.280)		(2.189)	(2.176)
$R_{t-12,t-1}$	0.844**		0.603*	0.702**	0.461		0.428	0.683
	(2.036)		(1.858)	(2.222)	(0.778)		(0.918)	(1.548)
$R_{t-36,t-13}$	12.344***		5.198***	5.263***	14.564***		6.019***	6.259***
	(13.477)		(8.119)	(8.419)	(12.165)		(6.786)	(7.226)
$size_t$		-8.040***	-7.801***	-8.111***		-12.714***	-12.372***	-12.029***
		(-15.357)	(-15.098)	(-15.087)		(-14.951)	(-14.620)	(-14.320)
$lvar_{t-1,t}$		1.575***	1.573***	1.564***		1.769***	1.756***	1.741***
		(13.399)	(13.359)	(13.281)		(9.238)	(9.247)	(9.206)
$Jump_t$		0.238***	0.215***	0.217***		0.380***	0.375***	0.374***
		(4.006)	(3.674)	(3.873)		(3.931)	(3.860)	(3.910)
Forward Controls	NO	NO	NO	YES	NO	NO	NO	YES
Intercept	-15.357***	-1.632	-4.032	-4.216	-14.359***	24.340***	21.368***	17.489**
	(-10.372)	(-0.375)	(-0.923)	(-0.954)	(-5.521)	(3.503)	(3.066)	(2.493)
# Observations	275,093	275,093	275,093	275,093	275,093	275,093	275,093	275,093
R^2	0.056	0.243	0.248	0.260	0.051	0.200	0.204	0.220

Table XI: ROE Cross-Sectional Regressions

Table XI reports mean coefficients and t-statistics based on the Fama and MacBeth (1973) approach (with a Newey-West correction) of one-period-ahead quarterly return on book equity, $roe_{t+1} = \log(1 + \frac{x_{t+1}}{B_t})$, on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on book equity. $\sigma^2_{t,t+1}$ represents future realized variance estimated by summing daily squared log returns. $\sigma^2_{QG,t,t+1}$ represents represents future realized variance estimated by summing daily conditional variances estimated using a QGARCH(1,1) model. $E_t[\sigma^2_{QG,t,t+1}]$ represents represents expected cumulative future variance calculated using the estimated on the parameters of a QGARCH(1,1) model. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
roe_t	0.620***			0.577***		0.537***		0.536***
	(45.329)			(43.086)		(41.345)		(40.562)
bm_t		-0.017***		-0.011***		-0.014***		-0.014***
		(-11.251)		(-11.510)		(-14.873)		(-14.174)
σ^2_{t+1}			-1.176***	-0.614***				
			(-19.712)	(-17.762)				
$\sigma^2_{QG,t,t+1}$					-1.565***	-0.882***		
					(-22.750)	(-19.883)		
$E_t[\sigma^2_{QG,t+1}]$							-1.205***	-0.675***
							(-30.443)	(-25.328)
Intercept	0.474***	0.383**	3.602***	0.585***	4.765***	1.118***	4.444***	0.928***
	(4.929)	(2.465)	(68.504)	(5.618)	(76.087)	(10.829)	(73.191)	(8.724)
# Observations	307,510	307,510	307,510	307,510	275,093	275,093	275,093	275,093
R^2	0.325	0.029	0.058	0.352	0.099	0.362	0.097	0.361